Quantum Null Energy Condition in Far-From-Equilibrium Systems

Based on work with Daniel Grumiller, Wilke van der Schee and Philipp Stanzer (1710.09837)

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  Holographic Calculation of QNEC
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Energy Conditions

Constraints on the energy-momentum tensor (EMT) which are necessary to proof certain theorems in general relativity.

name	inequality	energy and pressure
weak	$T_{ab}t^a t^b \geq 0$	$\rho > 0,\ \rho + p_i > 0$
null	$T_{ab}k^a k^b \geq 0$	$\rho + p_i \geq 0$
strong	$(T_{ab} - \frac{1}{2}T g_{ab})t^a t^b \geq 0$	$\rho + \sum_i p_i \geq 0,\ \rho + p_i \geq 0$
dominant	$-T^a_b t^b$ future directed	$\rho \geq 0,\ \rho \geq \lvert p_i \rvert$

Example: Focusing theorem ("matter focuses light") requires null energy condition (NEC) [Penrose 65]

Some energy conditions are already violated on the classical level.
Example: Minimally coupled scalar field violates the strong energy condition (SEC)

$$T_{ab}t^a t^b - \frac{1}{2}T = (t^a \nabla_a \phi)^2 - \frac{1}{2}m^2\phi^2$$

All classical energy conditions can be **violated by quantum matter fields**.

Question: Is there a **local** energy condition which also holds on the **quantum level**?

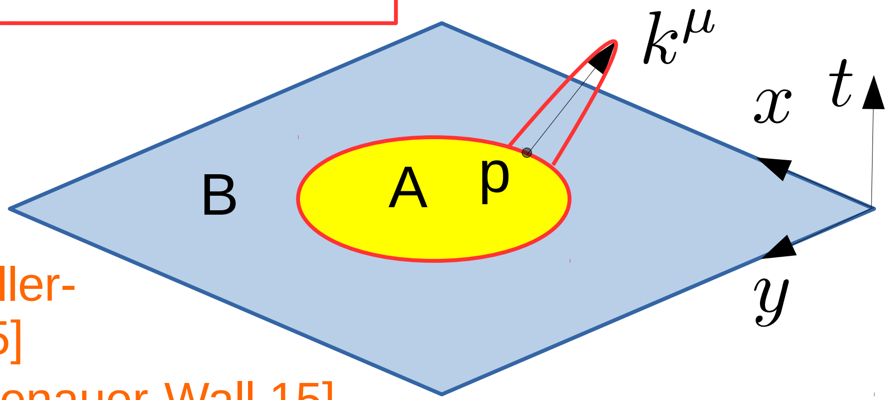
Quantum Null Energy Condition

The quantum null energy condition (QNEC) arose from the **Quantum Focusing Conjecture**, a semi-classical generalization of the Penrose focusing theorem.
[Bousso-Fisher-Leichenauer-Wall 15]

QNEC relates the null-projection of the EMT to the second variation of entanglement entropy with respect to a lightlike deformation of the entangling region.

$$\boxed{\langle T_{ab}(p) k^a k^b \rangle \geq \frac{\hbar}{2\pi\sqrt{h(p)}} \frac{\delta^2 S_{EE}}{\delta\lambda(p)^2}\Big|_{\lambda(p)=0} \quad \forall k^2(p)=0}$$

$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A \quad \rho_A = \mathrm{Tr}_B \rho$$



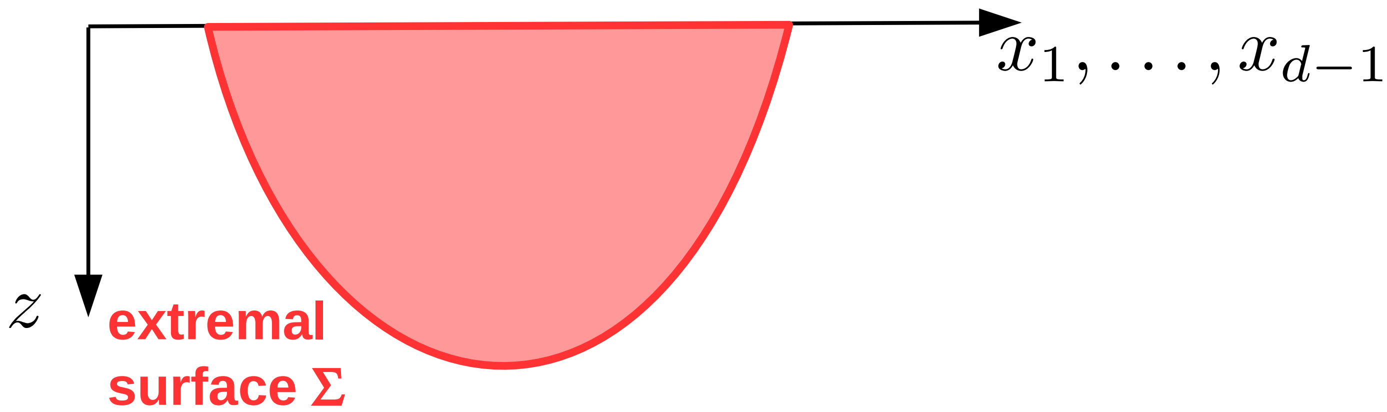
There exist several proofs:

- Free bosonic field theories [Bousso-Fisher-Koeller-Leichenauer-Wall 15]
- Holographic field theories [Bousso-Fisher-Leichenauer-Wall 15]
- General QFTs [Balakrishnan-Faulkner-Khandker-Wang 17]

QNEC in Holography

Entanglement entropy can be computed holographically form the **area of extremal surfaces** in the gravity theory. [Ryu-Takayanagi 06,Hubeny-Rangamani-Takayanagi 07]

$$S_{EE} = \frac{\mathcal{A}(\Sigma)}{4G_N}$$



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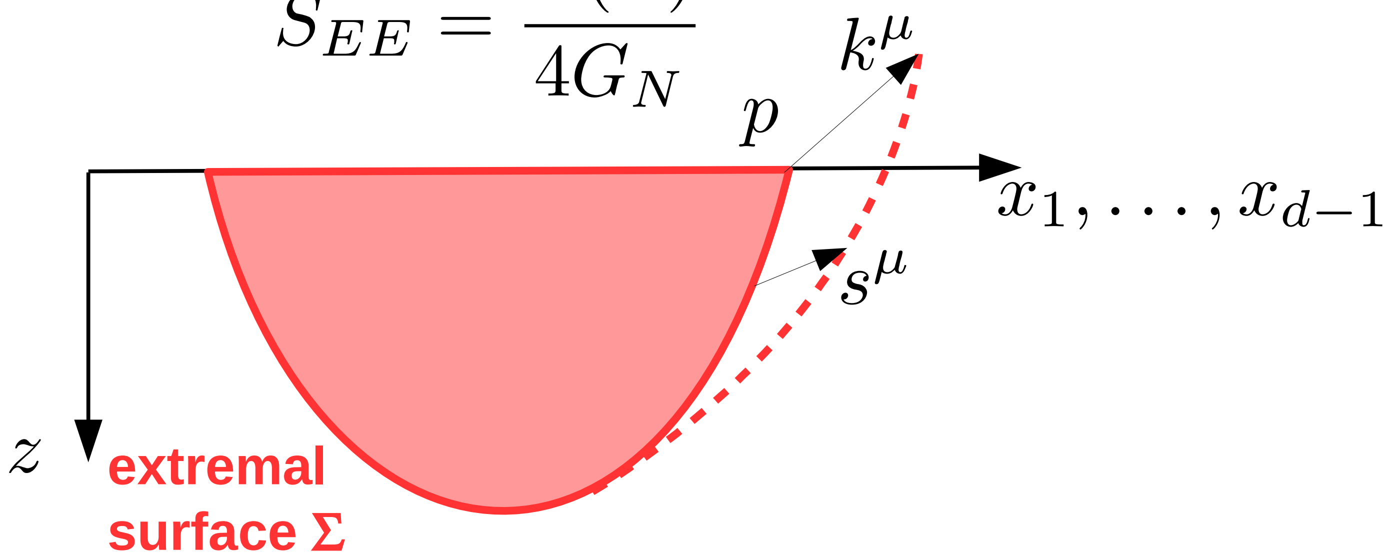


Geometric picture of QNEC: (NEC in bulk $\Longrightarrow$ "entanglement nesting property") [Wall 14]
Lightlike deformations in the boundary induce spacelike deformations of bulk surfaces.

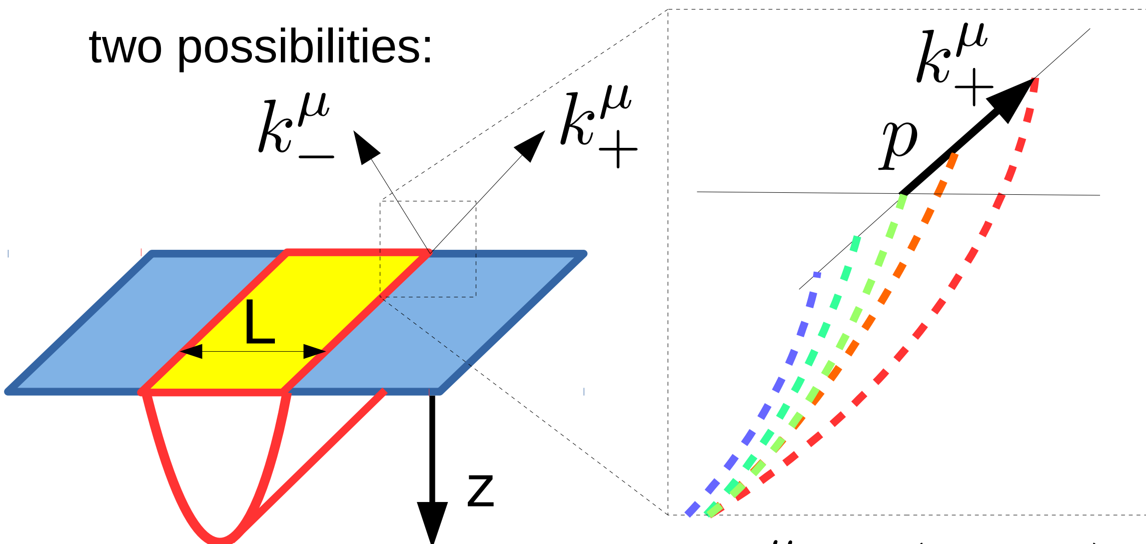
$$s_\mu s^\mu \geq 0 \quad \Leftrightarrow \quad \langle T_{kk}(p) \rangle \geq \frac{\hbar}{2\pi\sqrt{h}} S''_{EE}(p)$$

Holographic Computation of QNEC

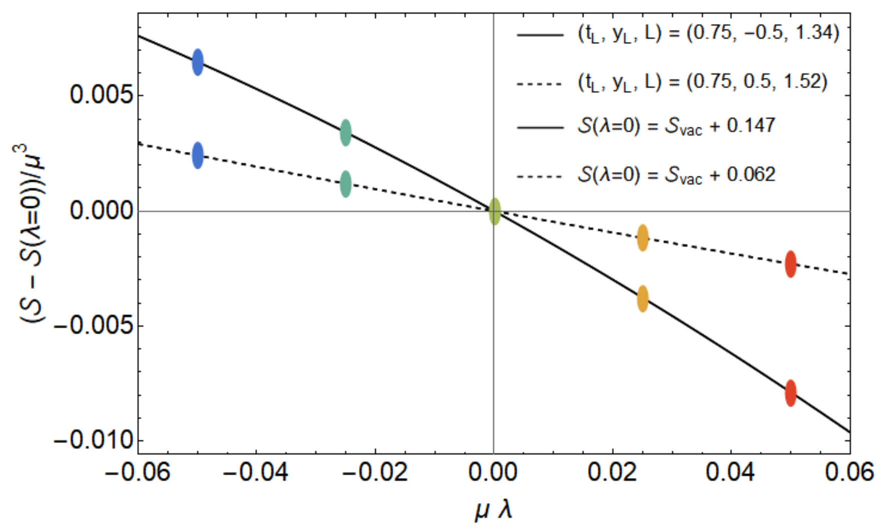
For simplicity, we consider entangling regions which are **infinite stripes** of finite width L.
Extremal surface equations reduce to to a **geodesic equation** in an **auxiliary spacetime**.

[CE-Grumiller-Stricker 15]

two possibilities:



$$k^\mu_\pm = (\lambda, \pm\lambda)$$



Our method:

1. Solve for the geometry, either numerically or (if possible) analytically.
2. Use **relaxation algorithm** to compute an array of deformed surfaces (areas) $\{\lambda_i, \mathcal{A}_i\}$.
3. Interpolating $\{\lambda_i, \mathcal{A}_i\}$ gives $\mathcal{A}(\lambda)$ from which we can compute the 2nd derivative.

$$S''_{EE} = \frac{1}{4G_N}\frac{d^2}{d\lambda^2}\mathcal{A}(\lambda)|_{\lambda=0}$$

Thermal Equilibrium State

Homogeneous thermal equilibrium state dual to AdS_5 Schwarzschild black brane.

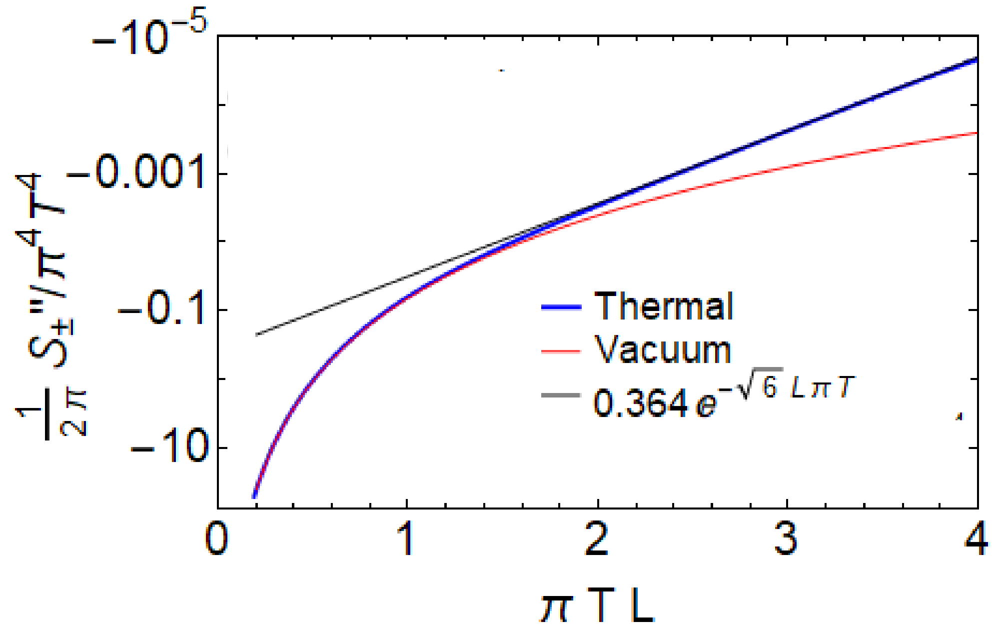
$$ds^2 = \frac{1}{z^2}\left(-dt^2 f(z) + dz^2/f(z) + dx^2\right)$$

$$f(z) = 1 - (\pi T)^2 z^4 \qquad k^\mu_\pm = \delta^\mu_t \pm \delta^\mu_x$$

$$\mathcal{T}_{\pm\pm} = \frac{1}{N_c^2}\langle T_{\mu\nu} k^\mu_\pm k^\nu_\pm \rangle = \frac{\pi^2}{2} T^4$$

parity invariance $\Rightarrow \mathcal{T}_{++} = \mathcal{T}_{--}$

time-reversal invariance $\Rightarrow S''_+ = S''_-$



- $S''_\pm < 0 \;\Rightarrow\;$ QNEC is **never saturated** and **always weaker** than NEC.
- **TL << 1:** $\frac{1}{2\pi} S''_\pm = \boxed{-\frac{1}{\pi^2 c_0^3 L^4}} + \frac{(\pi T)^4 c_0^3}{15\sqrt{3}\pi} - (\pi T)^8 L^4 \left(\frac{44 c_0^9}{675} - \frac{2 c_0^6}{3\pi^2}\right) + \mathcal{O}(T^{12} L^8) \quad c_0 = \frac{3\Gamma(1/3)^3}{2^{1/3}(2\pi)^2}$

  Vacuum solution (pure AdS, T=0)
- **TL >> 1:** $\frac{1}{2\pi} S''_\pm = -\frac{5\sqrt{6}\epsilon_0}{4\pi^2} (\pi T)^4 e^{-\sqrt{6} L (\pi T)} + \mathcal{O}(L e^{-2\sqrt{6} L}) \qquad \epsilon_0 \approx 1.173487$

Shock Wave Collisions

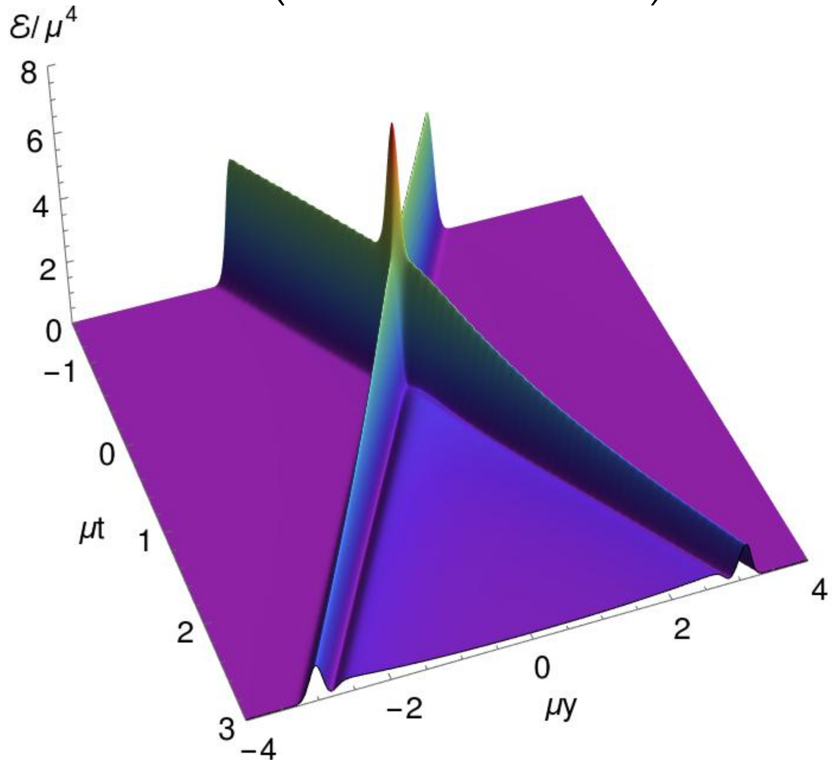
Collisions of planar sheets of strongly coupled N=4 SYM plasma on Minkowski.

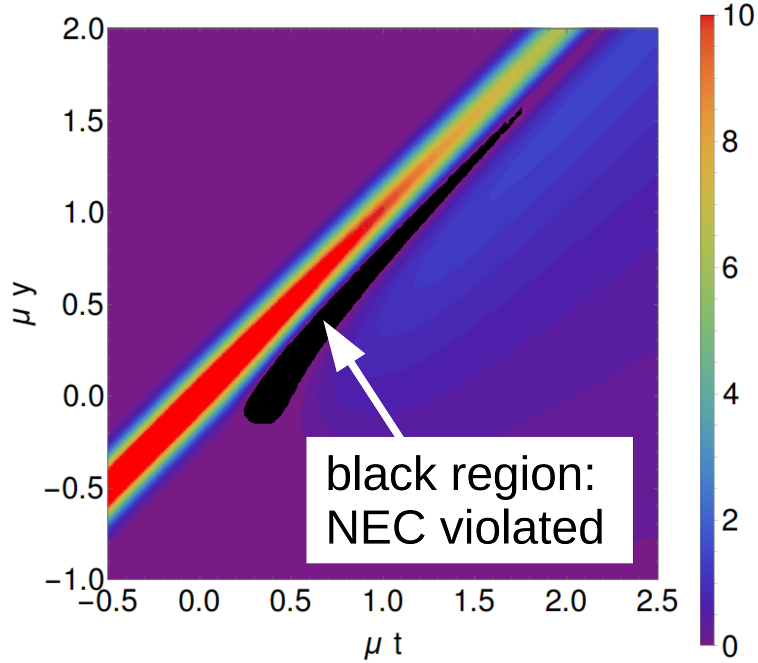
$$\langle T^{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\begin{pmatrix} \mathcal{E} & \mathcal{S} & 0 & 0 \\ \mathcal{S} & \mathcal{P}_\parallel & 0 & 0 \\ 0 & 0 & \mathcal{P}_\perp & 0 \\ 0 & 0 & 0 & \mathcal{P}_\perp \end{pmatrix}$$

time dependence $\Rightarrow S''_+ \neq S''_-$

no parity invariance $\Rightarrow \mathcal{T}_{++} \neq \mathcal{T}_{--}$

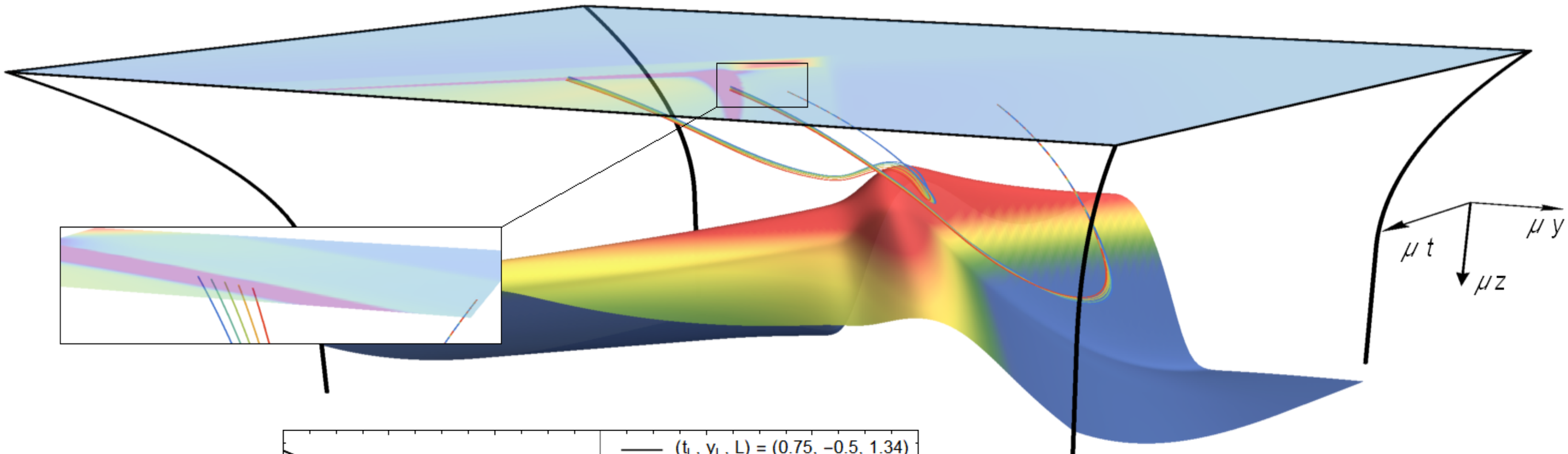
$$\frac{1}{N_c^2}\langle T_{\pm\pm}\rangle = \mathcal{E} \mp 2\mathcal{S} + \mathcal{P}_\parallel$$

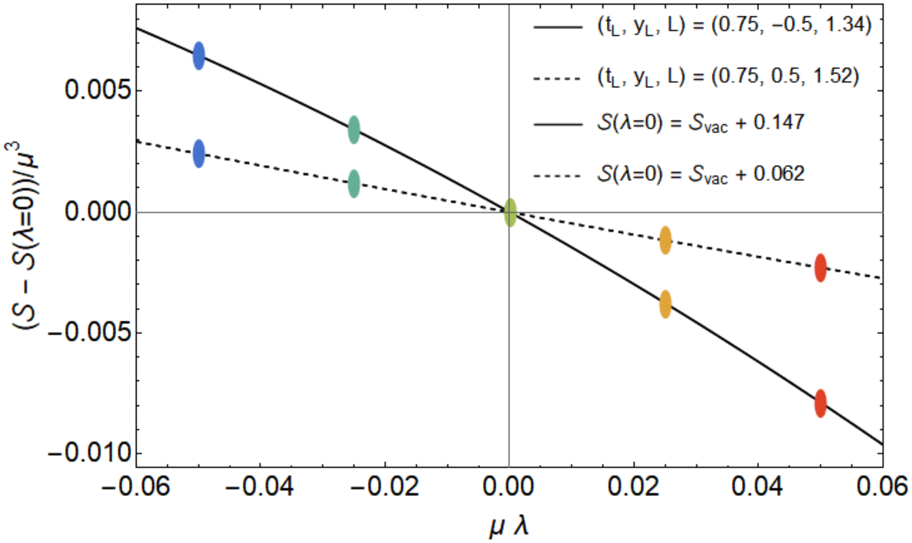




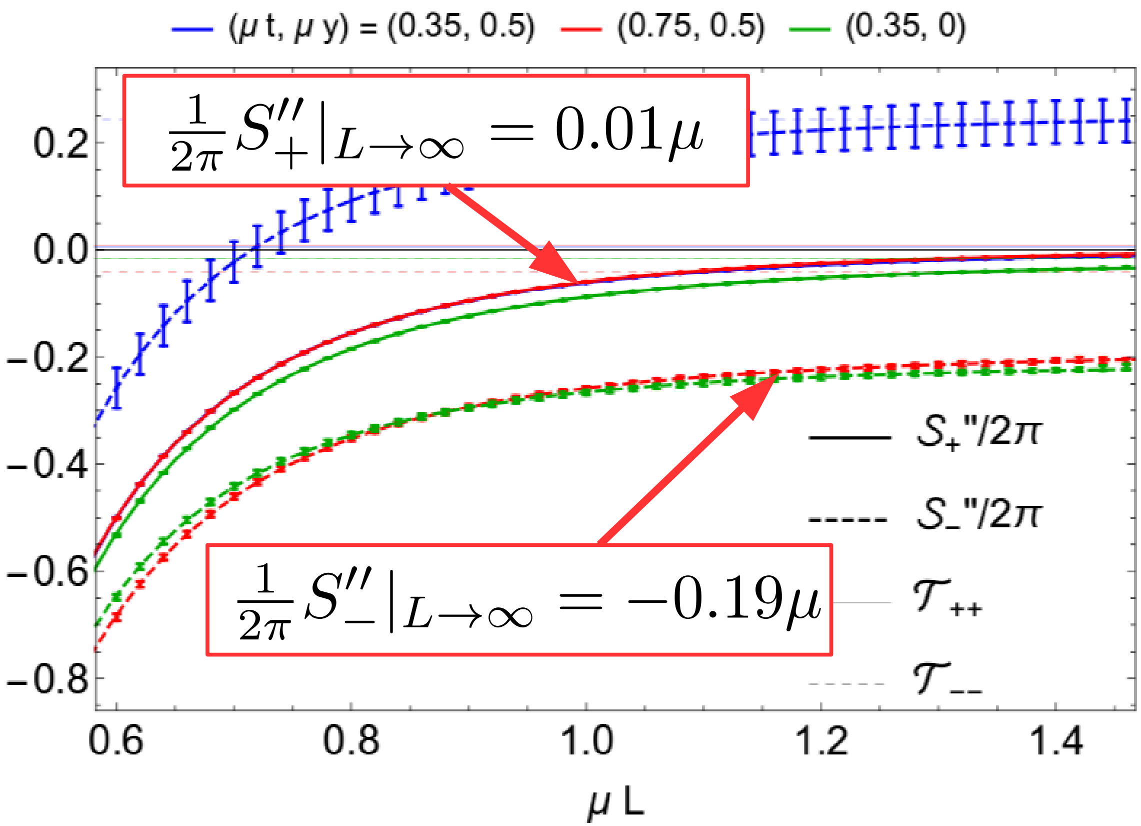
- Narrow shocks violate the classical null energy condition. [Arnold-Romatschke-Schee 14]

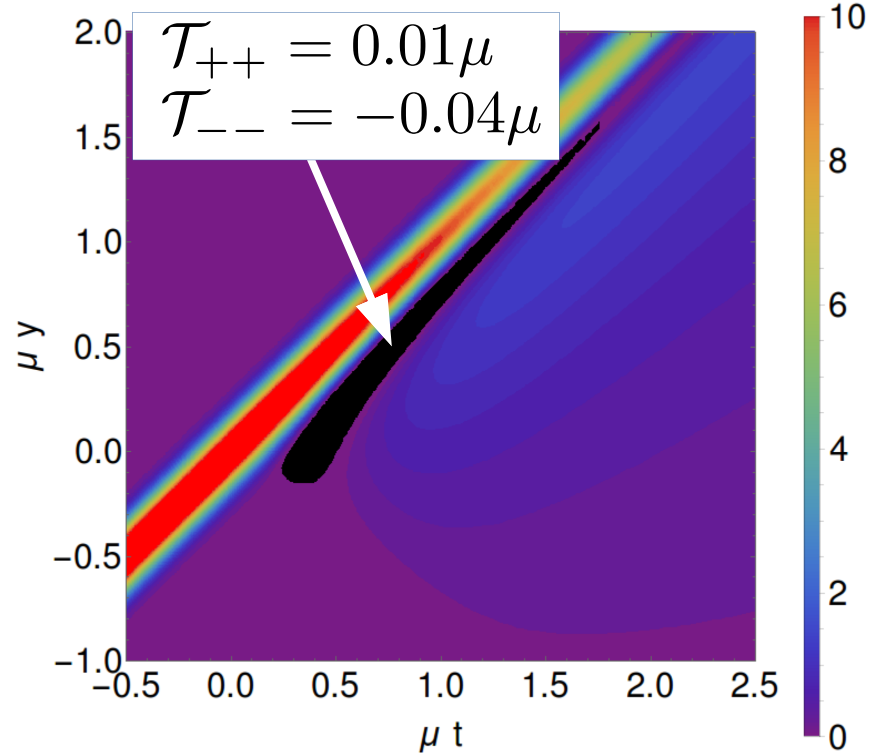
Extremal Surfaces in the Shock Wave Geometry





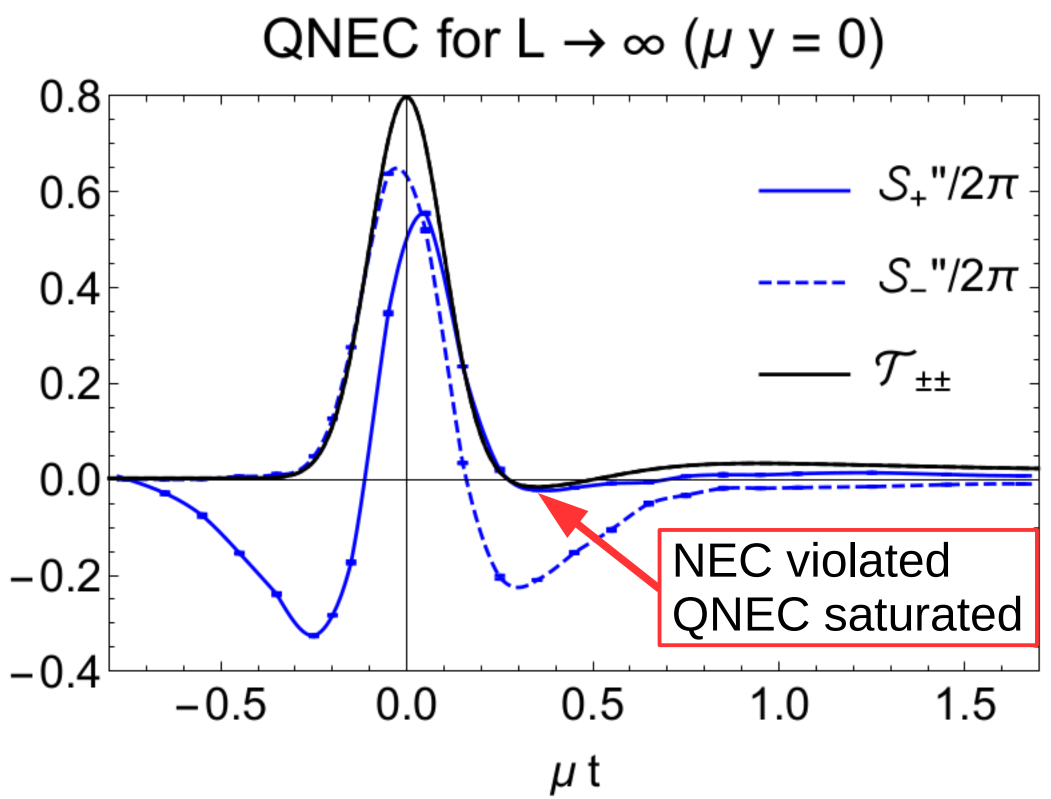
QNEC for Shock Waves (1)





- For example, at $(\mu t, \mu y) = (0.75, 0.5)$ the minus projection of **NEC is violated** but **QNEC holds** and is not saturated.
- At the same point the plus projection of NEC holds and QNEC saturates.

QNEC for Shock Waves (2)



- The large L limit of QNEC **saturates** in the **far-from-equilibrium** regime.
- At the parity invariant center (y=0) the minus projection of QNEC saturates shortly before the collision and the plus projection slightly after the collision.
- We find **no saturation** in the **hydrodynamic regime** $(\mu t > 0.8)$.

Summary

- The quantum null energy condition is a novel energy condition which is conjectured to hold in any QFT.
- We have seen the first series of explicit holographic calculations of QNEC in equilibrium and far-from-equilibrium systems.
- All our examples satisfy QNEC, which is a non-trivial check of the holographic proof.
- Surprising observation: In highly dynamic situations QNEC can be saturated, which typically seems not to be the case close to equilibrium.

Ongoing work

- BTZ black hole always saturates QNEC: $\langle T_{kk}\rangle = \frac{1}{2\pi}(S''_{EE} + \frac{1}{c}(S'_{EE})^2)$
- What happens at finite density, broken conformal symmetry, for different entangling regions, …? **Stay tuned!**

Far-From-Equilibrium Quench

Far-from-equilibrium quench dual to Vaidya AdS_5 geometry.
System evolves from vacuum ($t = -\infty$) to thermal equilibrium ($t = \infty$).

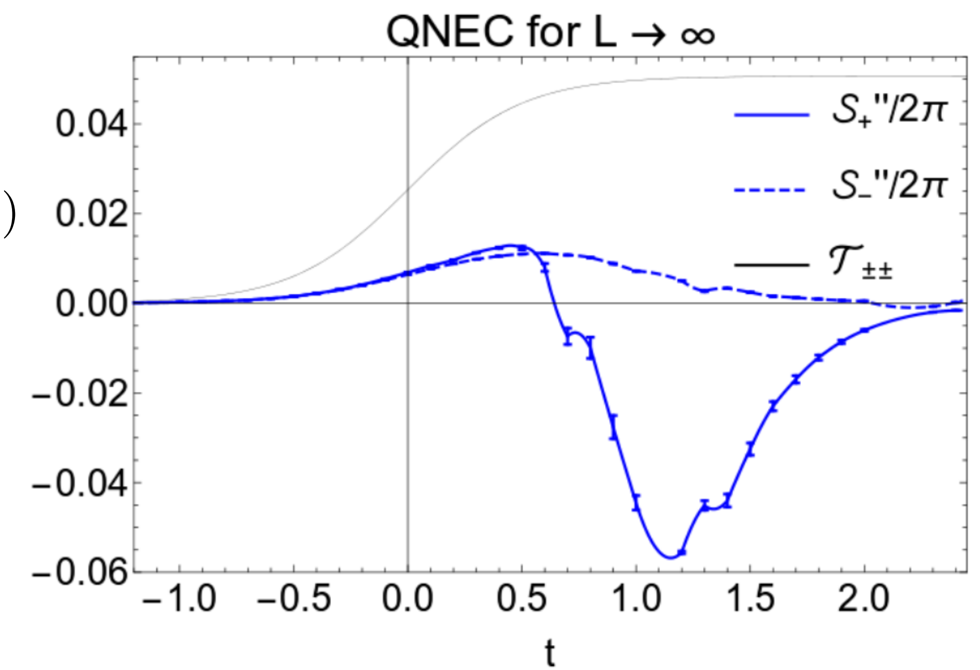
$$ds^2 = \frac{1}{z^2}(-dt^2 f(z) + dz^2/f(z) + dx^2)$$

$$f(z) = 1 - M(t)z^4 \quad M(t) = \tfrac{1}{2}(1 + \tanh(4t))$$

$$\mathcal{T}_{\pm\pm} = \frac{1}{N_c^2}\langle T_{\mu\nu} k_\pm^\mu k_\pm^\nu \rangle = \frac{1}{2\pi^2} M(t)$$

parity invariance $\Longrightarrow \mathcal{T}_{++} = \mathcal{T}_{--}$

time dependence $\Longrightarrow S_+ \neq S_-$



- Strongest version in the $L \to \infty$ limit.
- $S''_\pm$ can be positive $\Longrightarrow$ **QNEC is stronger than NEC**.
- QNEC **never saturates** (in our example): $S''_\pm$ mostly 30% of $\mathcal{T}_{\pm\pm}$.