Exploring nonlocal observables in shock wave collisions

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Outline

- Motivation

  QGP in heavy ion collisions

- Basics

  AdS/CFT, numerical GR on AdS, 2-point functions, entanglement entropy

- Results

  Time evolution of EMT, geodesics & extremal surfaces, 2-point functions, entanglement entropy

- Summary & Outlook

Motivation

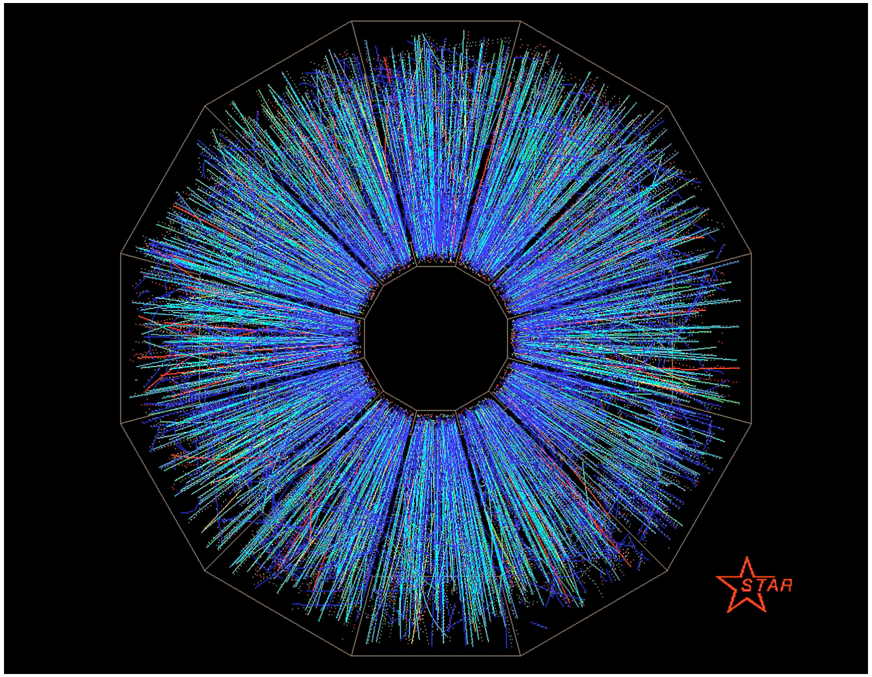
<u>Central question:</u>

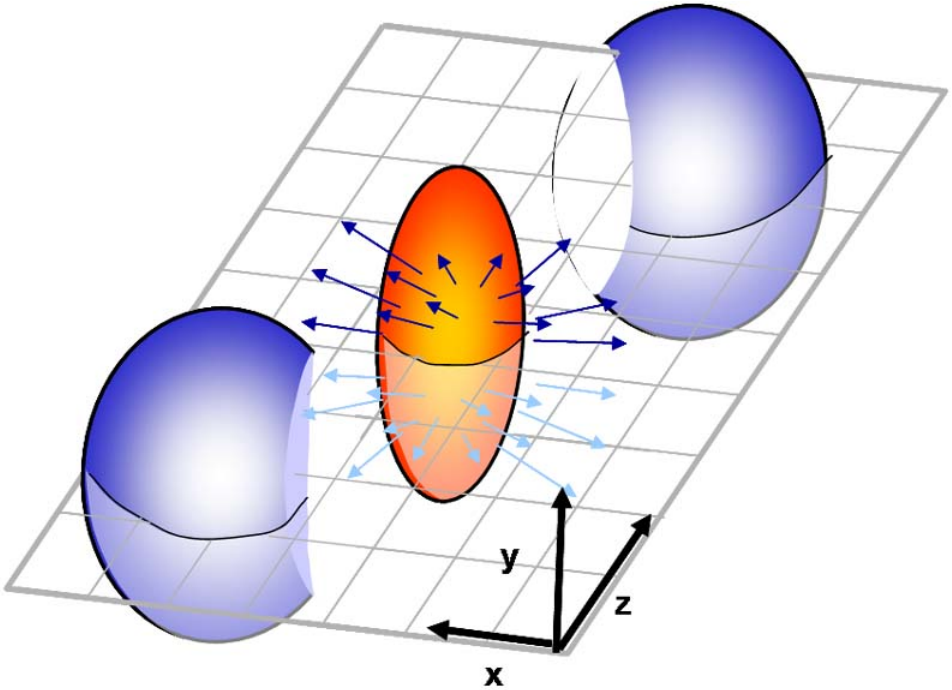
How does a **strongly coupled** quantum system which is initially **far-from equilibrium** evolve to its **equilibrium state?**

[picture: S. Schlichting]

Quark-gluon plasma in heavy ion collisions

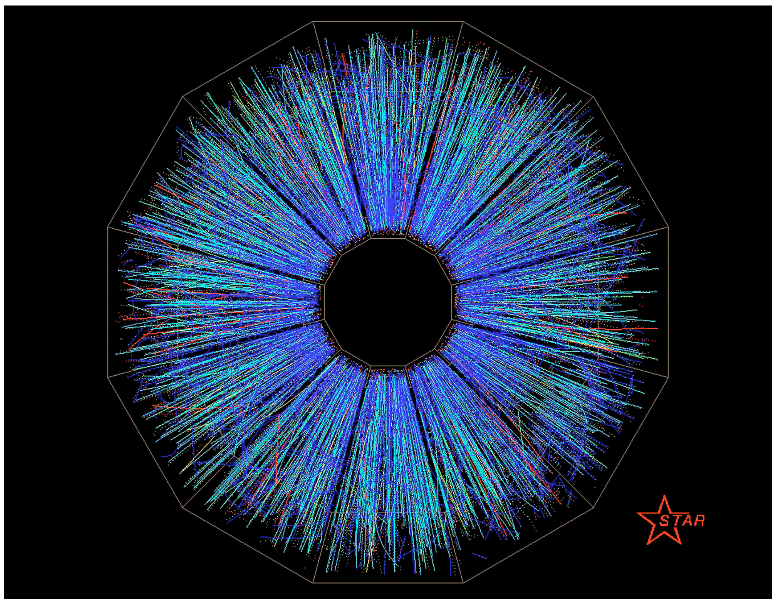
Quark-gluon plasma (QGP) is a **deconfined phase of quarks and gluons** produced in **heavy ion collision** (HIC) experiments at **RHIC** and **LHC**.

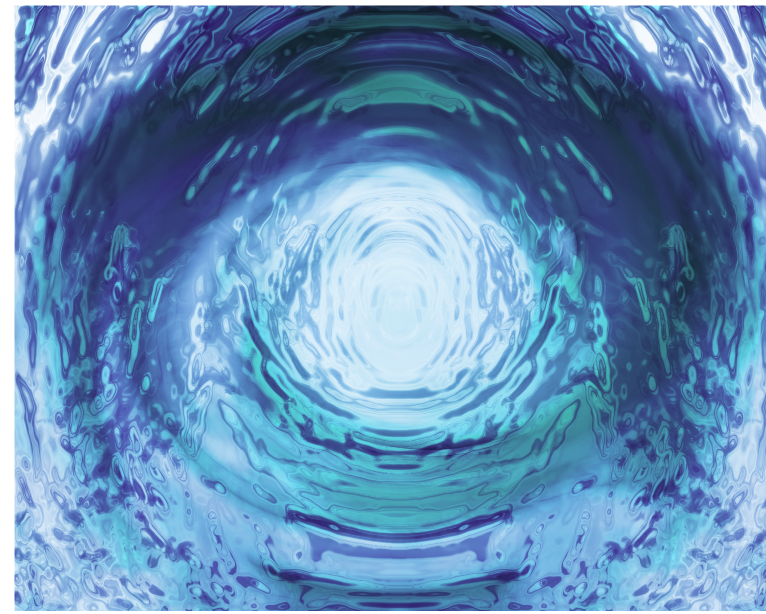




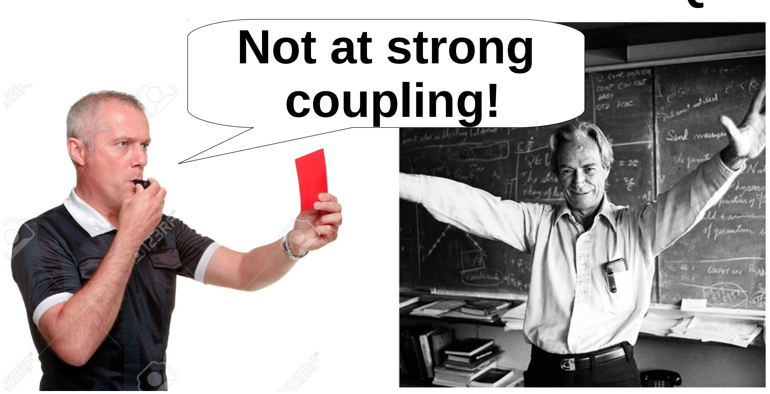
Why AdS/CFT?

The QGP produced in HIC's behaves like a **strongly coupled liquid** rather than a **weakly coupled gas.**

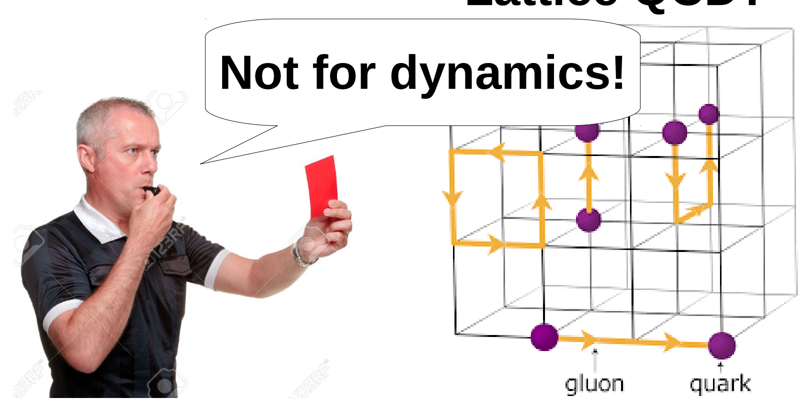




Perturbative QCD?

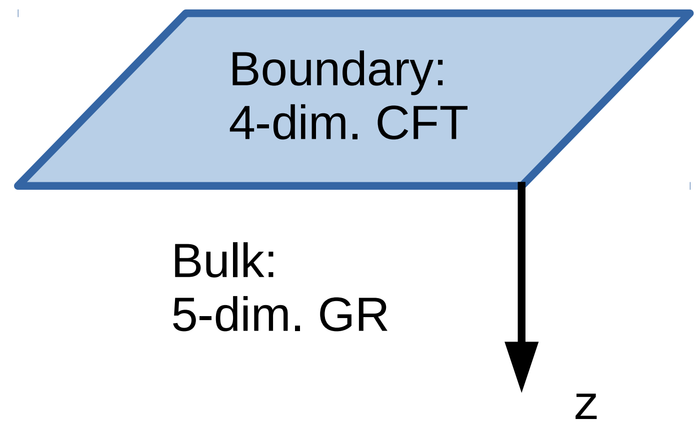


Lattice QCD?



AdS/CFT correspondence

AdS/CFT correspondence: [Maldacena 97]



Type IIB string theory on $AdS_5 \times S^5$ is equivalent to $\mathcal{N}$=4 super symmetric SU(N_c) **Yang-Mills theory** in 4D.

Supergravity limit:

Strongly coupled large N_c $\mathcal{N}$=4 SU(N_c) SYM theory is equivalent to **classical supergravity** on AdS_5

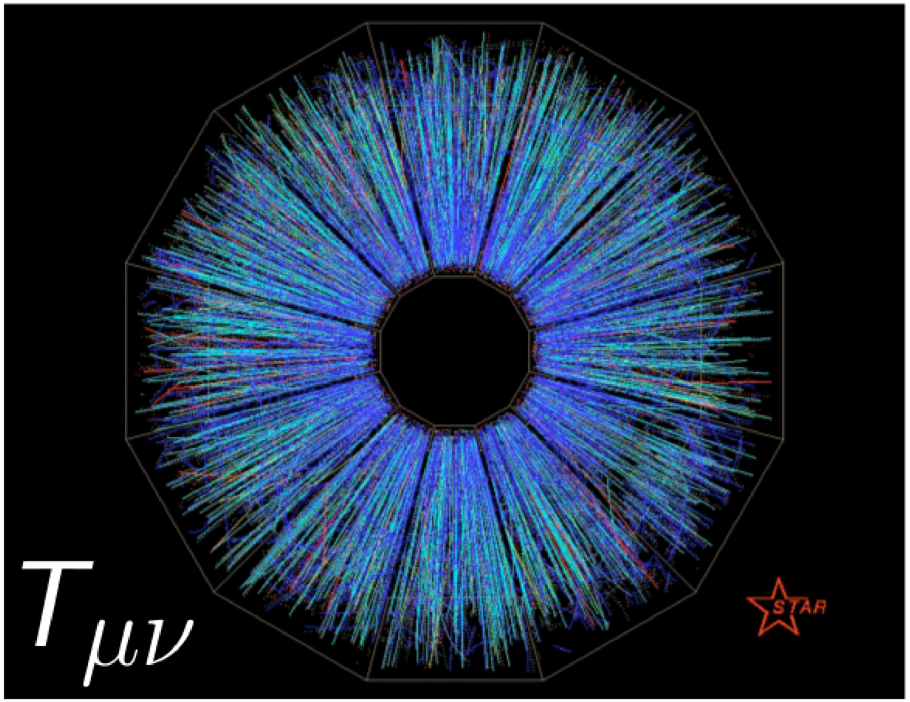
$$e^{-S_{\mathrm{sugra}}[\phi]}\Big|_{\lim\limits_{z\to 0}\phi=\phi_0} = \Big\langle \exp\Big(\int \mathrm{d}^d x \phi(x)\mathcal{O}(x)\Big)\Big\rangle_{CFT}.$$

Strategy:

- Use $\mathcal{N}$=4 SYM as **toymodel** for **QCD** in the strongly coupled regime.
- Build a **gravity model** dual to HICs, like colliding **gravitational shock waves**.
- Switch on the computer and solve the 5-dim. gravity problem **numerically**.
- Use the **holographic dictionary** to compute **observables in the 4 dim. field theory** form those gravity result.

Holographic thermalization

Thermalization = Black hole formation



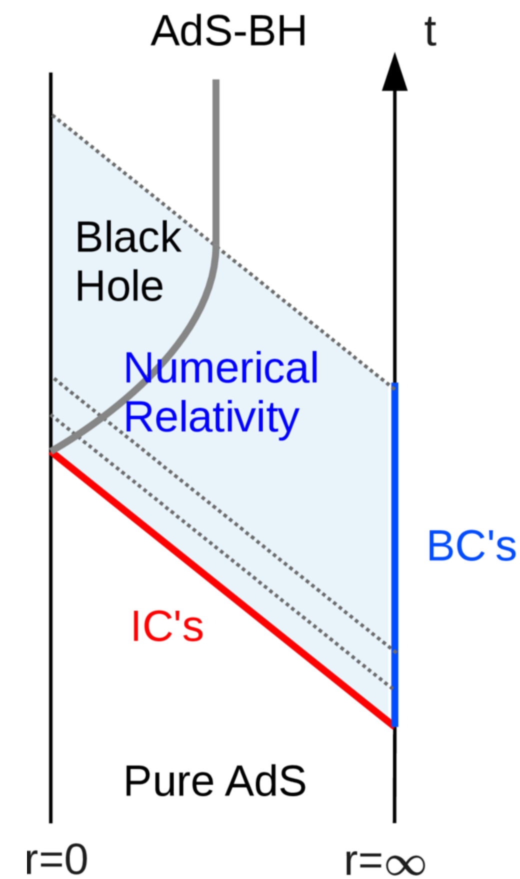
Solving time-dependent Einstein equations on asymptotically AdS

We want to solve the **5 dim. (vacuum) Einstein equations** (EE) with **negative cosmological constant Λ**

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0\,.$$

AdS is **not globally hyperbolic** – need **IC's & BC's** to formulate a **well defined initial value problem** (IVP).

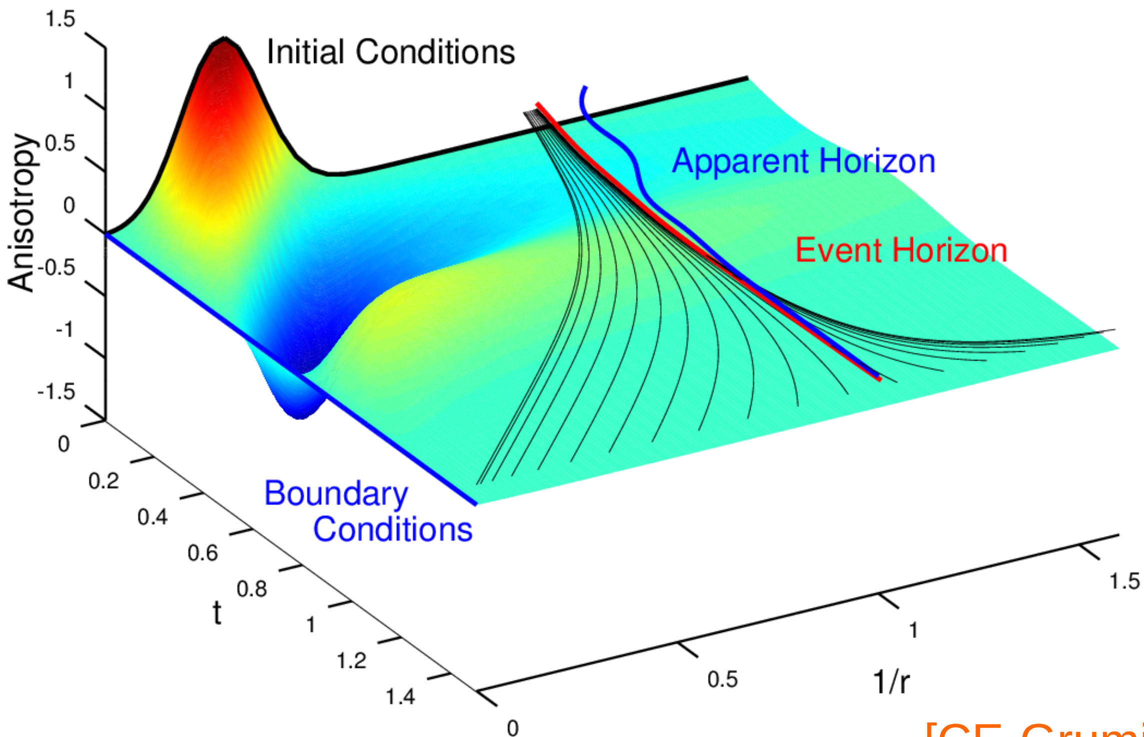
- **BC's:** boundary metric is **4-dim Minkovski**
  = **background metric** of the boundary **QFT**
- **IC's: two gravitational shock waves** in AdS
  = Lorentz contracted **nuclei in the QFT**

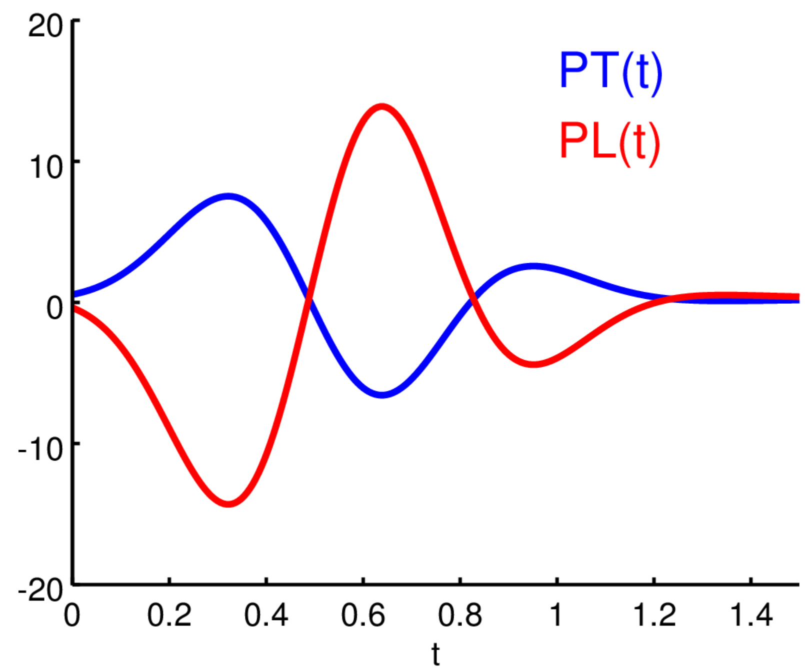


Isotropization of a homogeneous $\mathcal{N}$=4 SYM plasma

A **homogeneous** but **initially highly anisotropic** ($\mathcal{N}$=4 SYM) plasma **relaxates** to its **isotropic equilibrium state.** [Chesler-Yaffe 09]

The **dual gravity model** describes the **formation of a black brane** in an **anisotropic AdS$_5$** geometry.





[CE-Grumiller-Stricker 15]

Characteristic formulation

- We consider the following **homogeneous** and **anisotropic** ansatz for the metric in **Eddington-Finkelstein coordinates**

$$ds^2 = -A(r,v)dv^2 + 2dvdr + \Sigma(r,v)^2(e^{-2B(r,v)}dx_{\parallel}^2 + e^{B(r,v)}d\vec{x}_{\perp}^2)\,.$$

- **BC's:** We infer the **boundary metric** to be **conformally Minkovski**

$$ds^2|_{r\to\infty} = r^2(-dt^2 + d\vec{x}^2)\,.$$

- In these coordinates the **Einstein equations decouple** into a **nested set of ODEs**

IC's: $B_{v=v_0} \longrightarrow$

$$\Sigma'' + \tfrac{1}{2}B'^2 = 0 \quad (1)$$

$$\Sigma(\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2 = 0 \quad (2)$$

$$\Sigma(\dot{B})' + \tfrac{3}{2}(\Sigma'\dot{B} + 2B'\dot{\Sigma}) = 0 \quad (3)$$

$$A'' + 3B'\dot{B} - 12\Sigma'\dot{\Sigma}/\Sigma^2 + 4 = 0 \quad (4)$$

$$\ddot{\Sigma} + \tfrac{1}{2}(\dot{B}^2 - A'\dot{\Sigma}) = 0 \quad (5)$$

Constraint

$\xrightarrow{A}$ $\dot{B} = \partial_v B + \frac{1}{2}A\partial_r B$ $\xrightarrow{\partial_v B}$ $B_{(v+\Delta v)} = B_{(v)} + \Delta v \partial_v B_{(v)}$ $\longrightarrow (1)$

- At each $v = const.$ slice we **solve the ODEs** with a **spectral method**.
- For the **time evolution** we use the **4th order Runge-Kutta method**.

Field redefinitions

The **previous formulas** can be written down nicely in a paper but they are **not very useful for a numerical treatment**.

- **Residual gauge freedom** $r \to r + \xi(v)$ can be useful to fix the position of the horizon.
- The **inverse radial coordinate** $z = \frac{1}{r}$ transforms the AdS-boundary to $z = 0$.
- The following **redefinitions** give **finite metric functions** suitable for numerics

$$A(z,v) \to \frac{1}{z^2} + zA(z,v)\,, \quad \Sigma(z,v) \to \frac{1}{z} + z^2\Sigma(z,v)\,, \quad B(z,v) \to z^3 B(z,v)\,.$$

- From the new fields one can **directly read of** the components of the **EMT**

$$b_4(t) = B'(0,t)\,, \qquad a_4 = A'(0,t)\,,$$

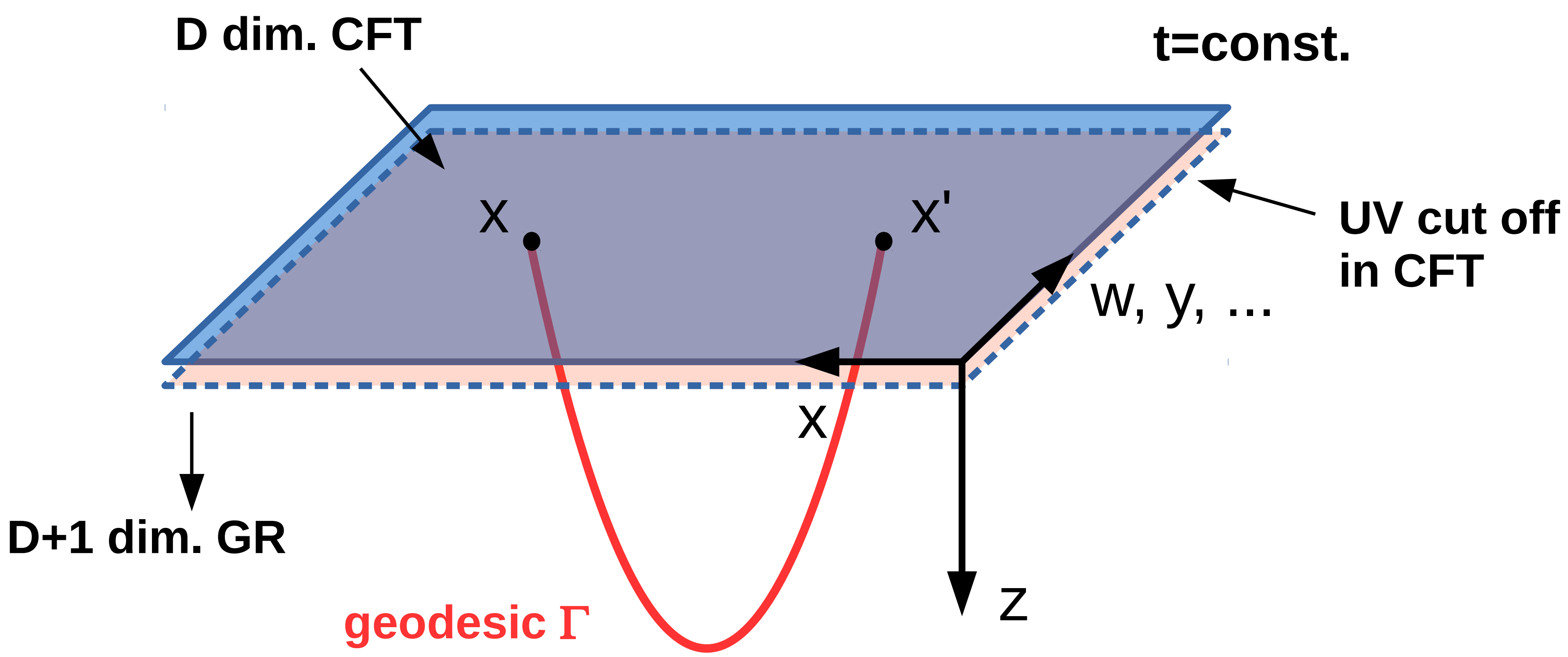
$$\mathcal{E} = -\frac{3}{4}a_4\,, \quad \mathcal{P}_{\parallel}(t) = -\frac{1}{4}a_4 - 2b_4(t)\,, \quad \mathcal{P}_{\perp}(t) = -\frac{1}{4}a_4 + b_4(t)\,,$$

$$\langle T^{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\mathrm{diag}[\mathcal{E}, \mathcal{P}_{\parallel}(t), \mathcal{P}_{\perp}(t), \mathcal{P}_{\perp}(t)]\,.$$

Two point functions

Equal time **2-point functions** for operators $\mathcal{O}(t,x)$ of **large conformal weight Δ** can be computed form the **length of geodesics**. [Balasubramanian-Ross 00]

$$\langle \mathcal{O}(t,x)\mathcal{O}(t,x')\rangle = \int \mathcal{D}\mathcal{P} e^{i\Delta\mathcal{L}(\mathcal{P})} \approx \sum_{\text{geodesics}} e^{-\Delta L_g} \approx e^{-\Delta L}$$



Entanglement entropy

Divide the system into **two parts** A,B.
The total Hilbert space factorizes:

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

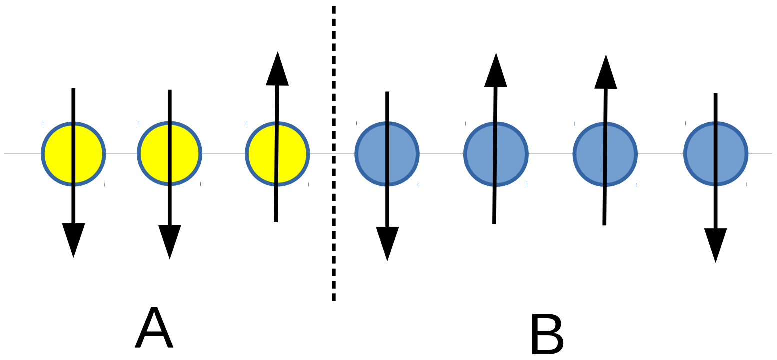
The **reduced density matrix** of A is obtained by the trace over $\mathcal{H}_{\mathcal{B}}$

$$\rho_A = \mathrm{Tr}_B \rho$$

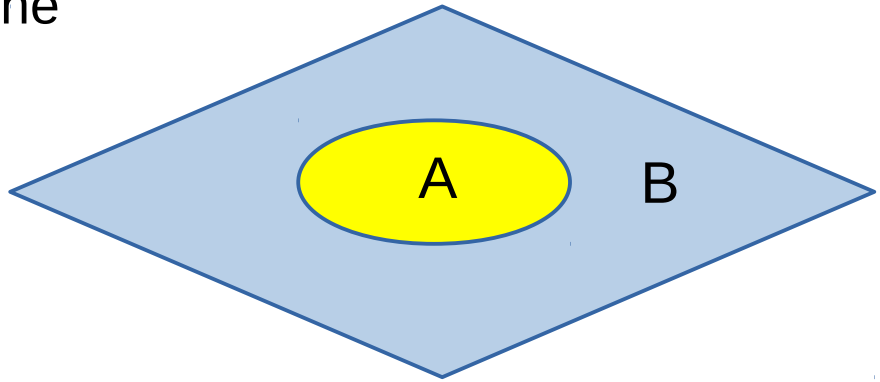
Entanglement entropy is defined as the **von Neumann entropy** of ρ_A:

$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$

lattice model



quantum field theory



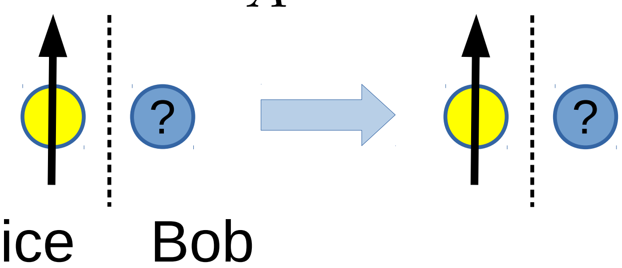
Entanglement entropy in a two quantum bit system

Consider a quantum system of two spin 1/2 dof's.
Observer Alice has only access to one spin and Bob to the other spin.

A **product state (not entangled)** in a two spin 1/2 system:

$$S_A = 0$$

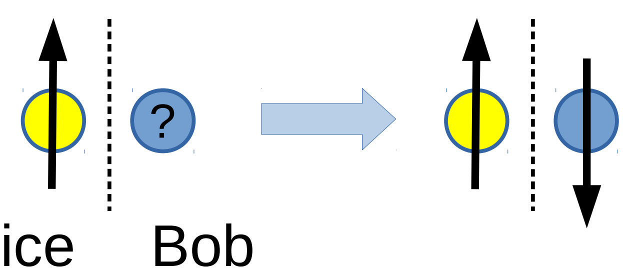
$$|\psi\rangle = \frac{1}{2}(|\uparrow_A\rangle + |\downarrow_A\rangle) \otimes (|\uparrow_B\rangle + |\downarrow_B\rangle)$$



A (maximally) **entangled state** in a two spin 1/2 system:

$$S_A = \log 2$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A\rangle \otimes |\downarrow_B\rangle - |\downarrow_A\rangle \otimes |\uparrow_B\rangle)$$



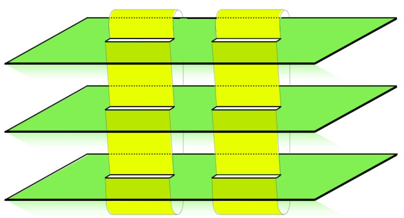
Entanglement entropy is a **measure** for **entanglement** in a quantum system.

Entanglement entropy in quantum field theories

The Basic Method to compute entanglement entropy in quantum field theories is the **replica method.**

Involves path integrals over n-sheeted Riemann surfaces ~ it's **complicated!**

With the **replica method** one gets **analytic results** for **1+1 dim. CFTs**. [Holzhey-Larsen-Wilczek 94]



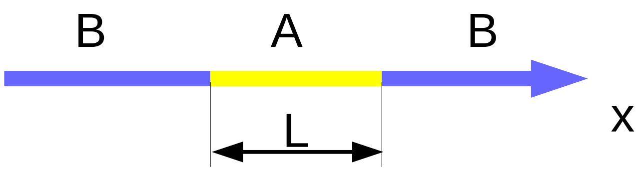
3-sheeted Riemann surface

One finds **universal scaling** with interval size:

central charge of the CFT

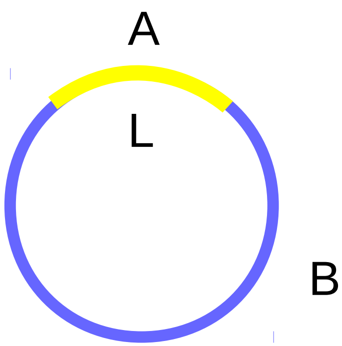
$$S_A = \frac{c}{3}\log\frac{L}{a} + finite$$

UV cut off



Notable generalization: 1+1 dim. Galilean CFTs [Bagchi-Basu-Grumiller-Riegler 15]

1+1 dim. CFTs



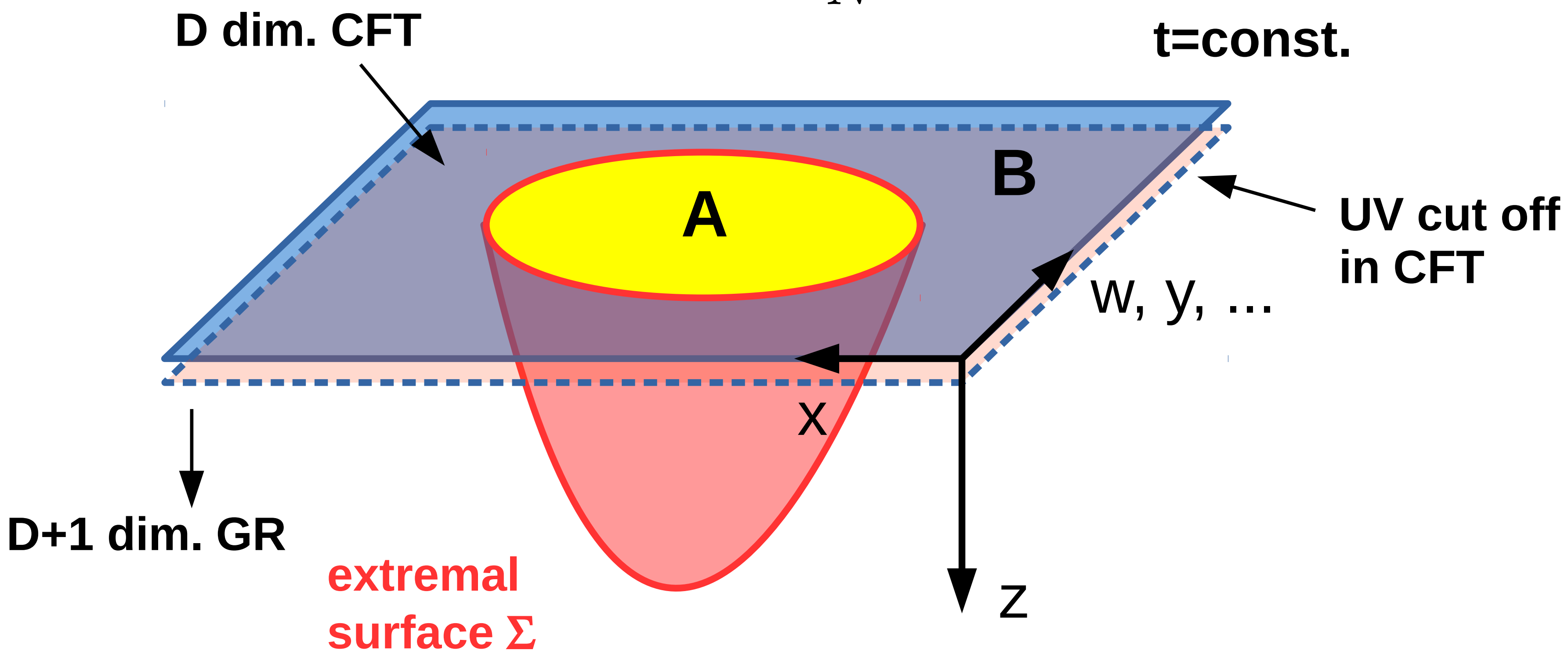
AdS/CFT provides a **simpler method** that works also in **higher dimensions**.

Holographic entanglement entropy

Within **AdS/CFT** entanglement entropy can be computed form the **area of minimal (extremal) surfaces** in the gravity theory.

$$S_A = \frac{\text{Area}(\Sigma)}{4G_N}$$

[Ryu-Takayanagi 06, Hubeny-Rangamani-Takayanagi 07]

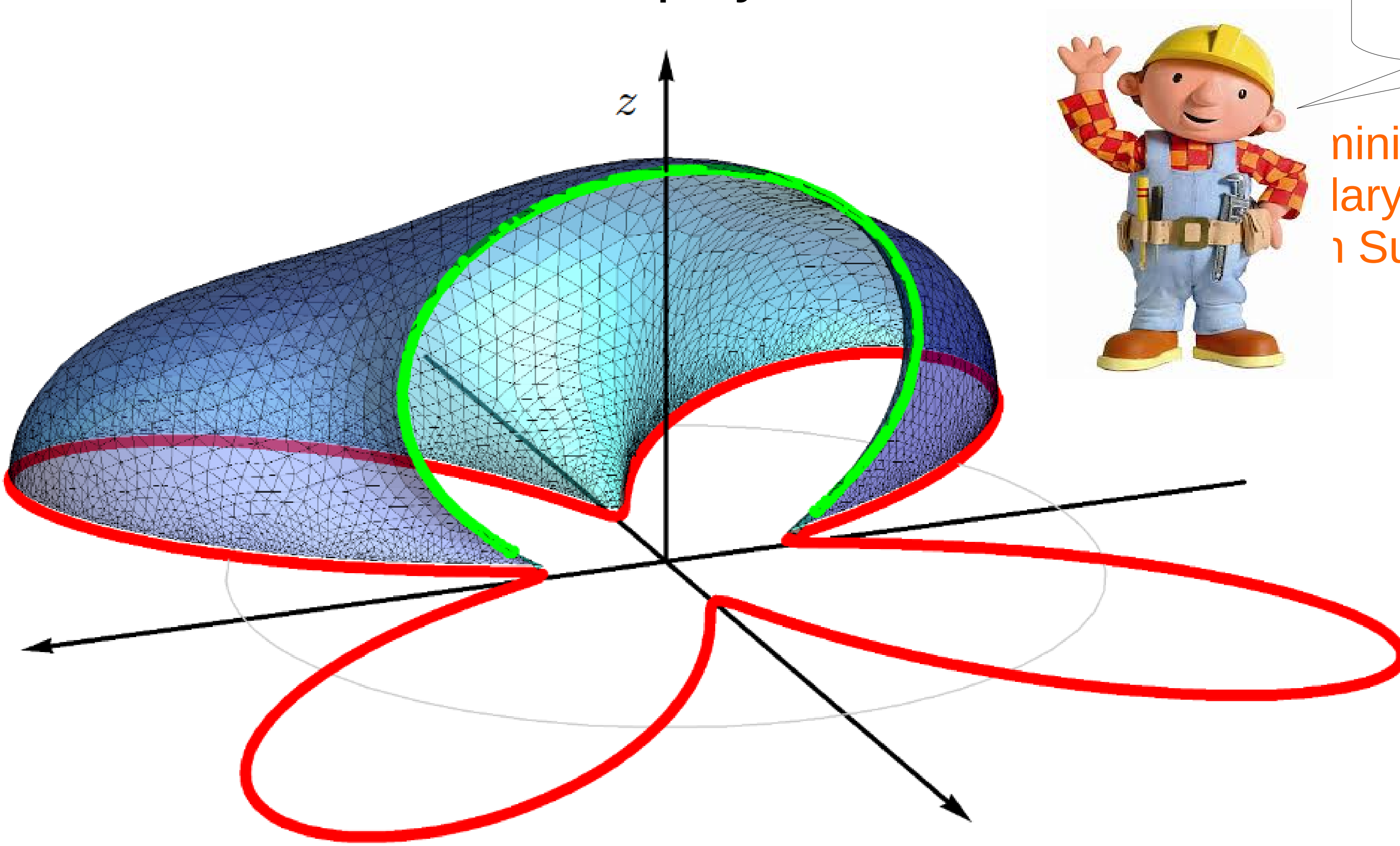


Holographic entanglement entropy

- In practice computing extremal co-dim. 2 hyper-surfaces is numerically involved. [work in progress: CE-Grumiller-Khavari]
- Can we somehow simplify our lives?

Yes we can!

ninimal surface for a star
lary region (red) in AdS4
ı Surface Evolver]

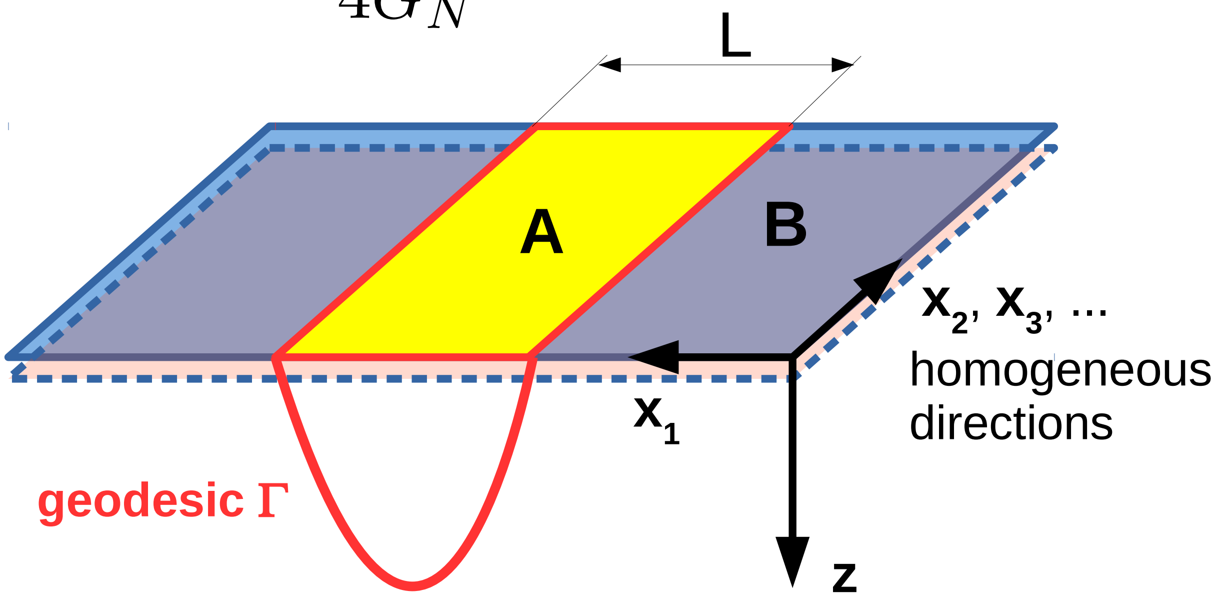


Entanglement entropy from geodesics

Consider a **stripe region** of **infinite extend** in **homogeneous directions** of the geometry. The **entanglement entropy** is prop. to the **geodesics length** in an **auxiliary spacetime**.

$$\mathcal{A} = \int d^3\sigma \sqrt{\det\left(\frac{\partial X^\mu}{\partial \sigma^a}\frac{\partial X^\nu}{\partial \sigma^b} g_{\mu\nu}\right)} = \int dx_3 \int dx_2 \int d\sigma \sqrt{\Omega^2 g_{\mu\nu}\frac{\partial X^\mu}{\partial \sigma}\frac{\partial X^\nu}{\partial \sigma}}$$

$$S_A = const. \frac{\text{Length}(\Gamma)}{4G_N} \qquad \tilde{g}_{\mu\nu} = \Omega(z,t,x_1)^2 g_{\mu\nu}$$

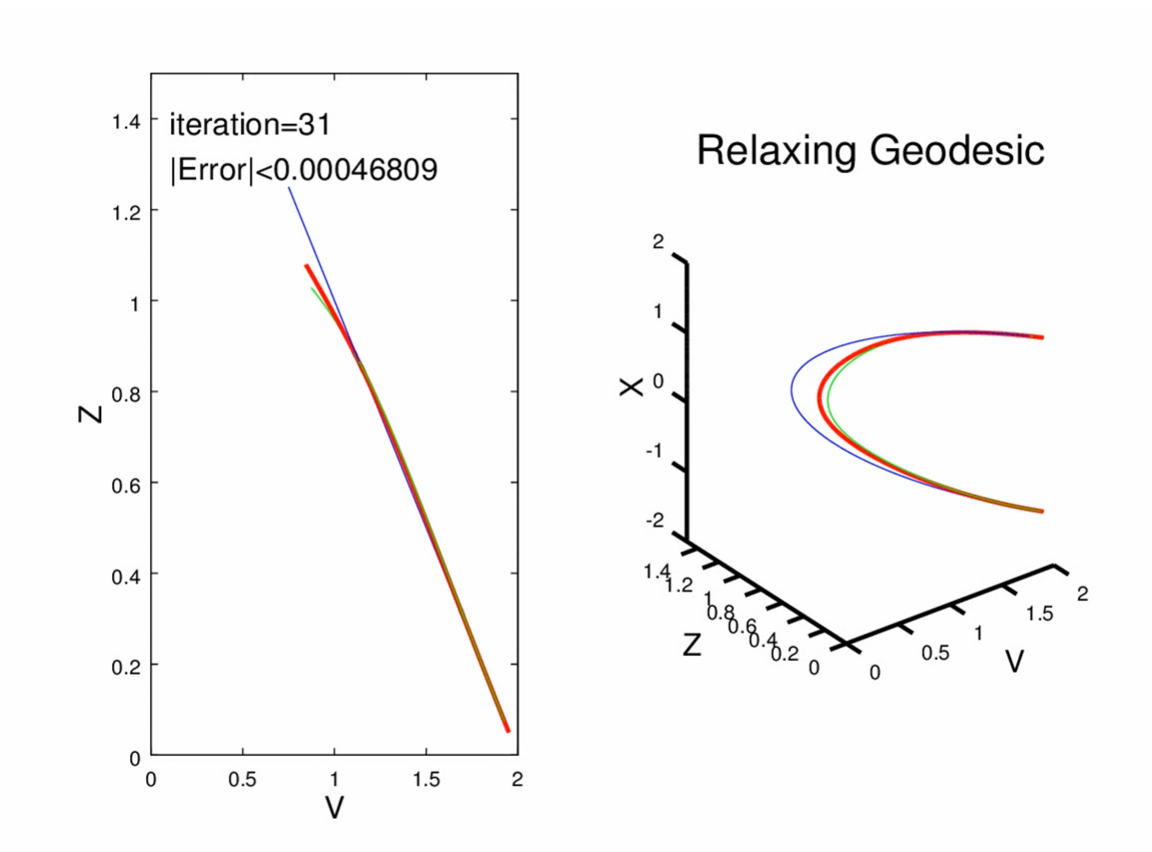


Numerics: relax, don't shoot!

Geodesic equation as **two point boundary value problem:**

$$\ddot{X}^{\mu} + \Gamma^{\mu}_{\alpha\beta}\dot{X}^{\alpha}\dot{X}^{\beta} = -J\dot{X}^{\mu}\,,$$

BC's: $X(\pm 1)^{\mu} \equiv (V(\pm 1), Z(\pm 1), X(\pm 1)) = (t, 0, \pm l/2)$



- There are two **standard numerical methods** for solving two point boundary value problems: [see Numerical Recipes]

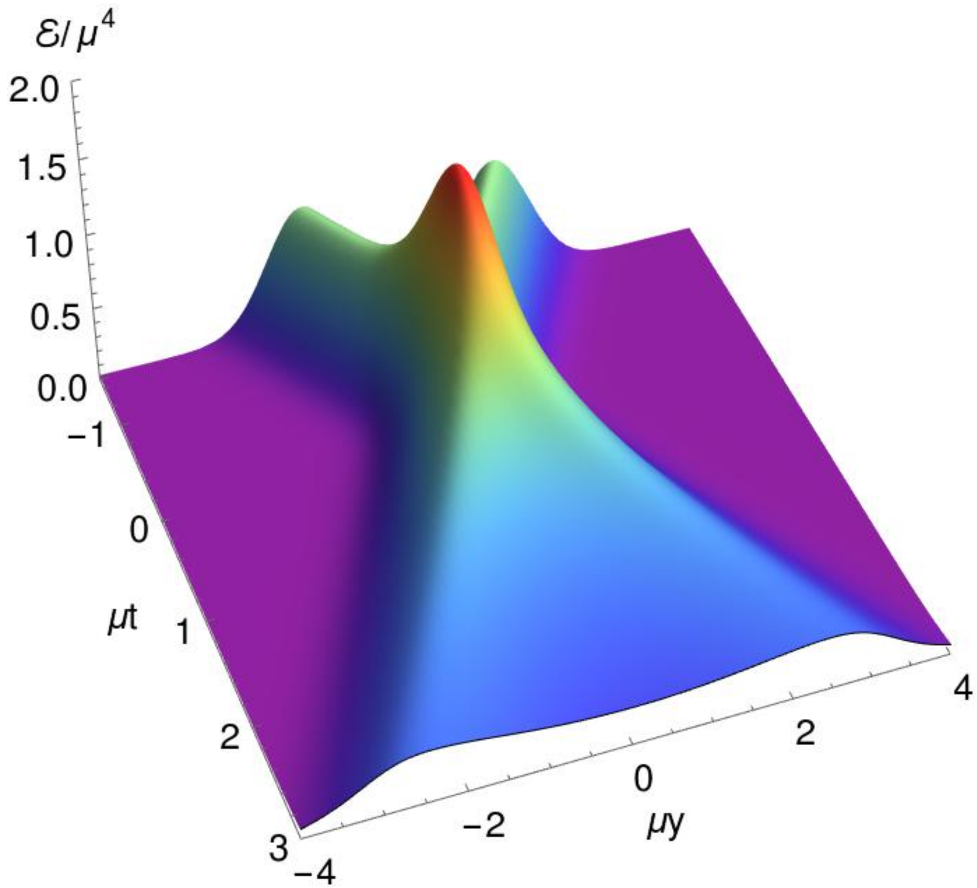
Shooting:
Very **sensitive to initialization** on **asymptotic AdS** spacetimes.

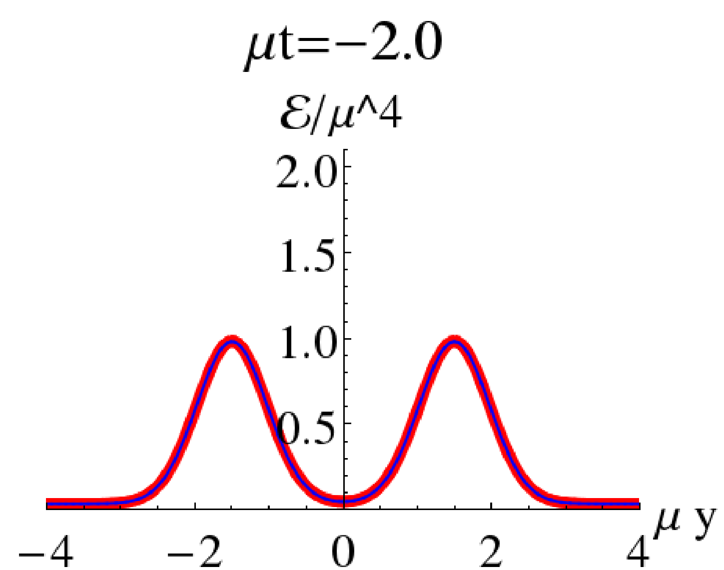
Relaxation:
Converges very **fast** if **good initial guess** is provided.

Holographic shock wave collisions

HIC is modeled by **two colliding sheets of energy** with **infinite extend in transverse direction** and **Gaussian profile** in **beam direction**. [Chesler-Yaffe 10]

$$ds^2 = -A(r,v,y)dv^2 + 2dv(dr + F(r,v,y)dy) + \Sigma(r,v,y)^2(e^{-2B(r,v,y)}dy^2 + e^{B(r,v,y)}d\vec{x}^2)$$





Initial conditions

- The pre-collision geometry describing two shocks moving in $\pm\tilde{y}$-direction in Fefferman-Graham coordinates $(\tilde{r},\tilde{t},\tilde{y},\tilde{\vec{x}})$ can be written down explicitly

$$ds^2 = \tilde{r}^2 \eta_{\nu\mu} d\tilde{x}^\mu d\tilde{x}^\nu + \tfrac{1}{\tilde{r}^2}\left(d\tilde{r}^2 + h(\tilde{t}+\tilde{y})(d\tilde{t}+d\tilde{y})^2 + h(\tilde{t}-\tilde{y})(d\tilde{t}-d\tilde{y})^2\right) \,.$$

- The function $h(\tilde{t}\pm\tilde{y})$ is an arbitrary function for which we choose a Gaussian

$$h(\tilde{t}\pm\tilde{y}) = \frac{\mu^3}{\sqrt{2\pi\omega^2}} e^{-\frac{(\tilde{t}\pm\tilde{y})^2}{2\omega^2}} \,.$$

- In this gauge the EMT describes two lumps of energy with maximum overlap at $\tilde{t}=0$

$$\tilde{T}^{\tilde{t}\tilde{t}} = \tilde{T}^{\tilde{y}\tilde{y}} = h(\tilde{t}-\tilde{y}) + h(\tilde{t}+\tilde{y})\,, \qquad \tilde{T}^{\tilde{t}\tilde{y}} = h(\tilde{t}-\tilde{y}) - h(\tilde{t}+\tilde{y})\,.$$

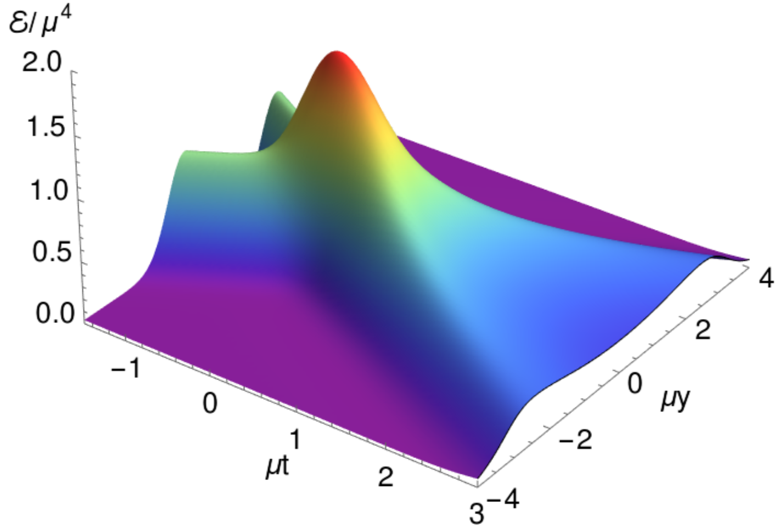
- For the time evolution these initial conditions need to be (numerically) transformed to Eddington-Finkelstein gauge.

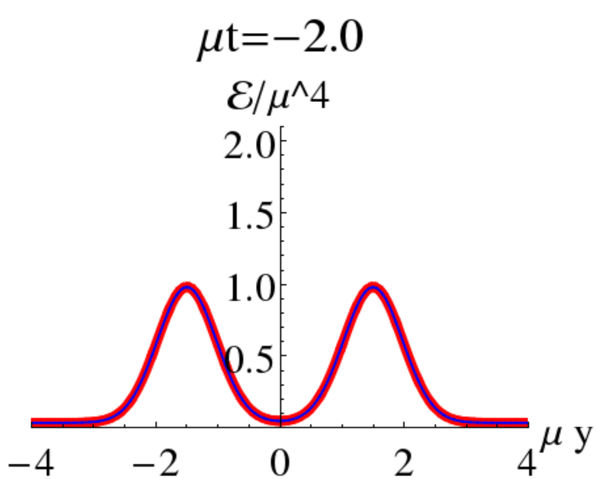
Wide vs. narrow shocks

[Solana-Heller-Mateos-van der Schee 12]

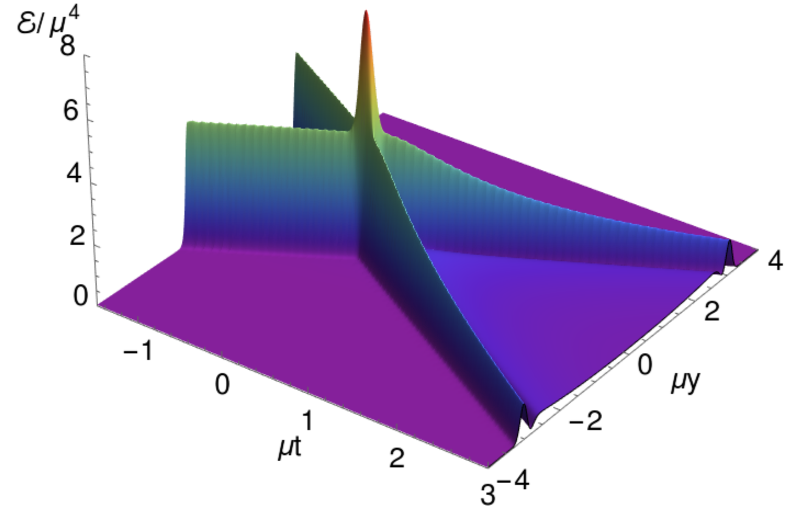
Two qualitatively different dynamical regimes

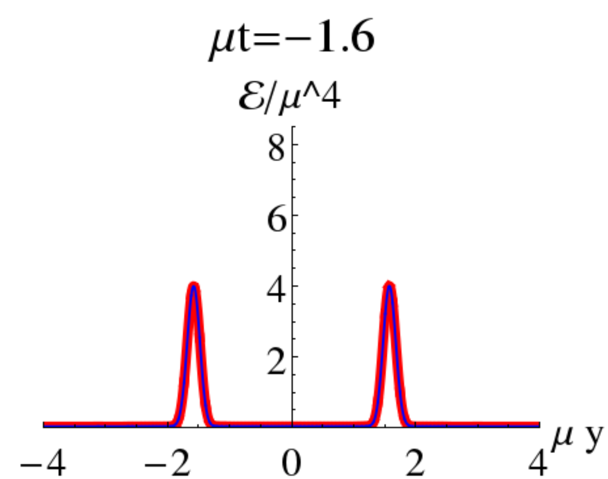
- **Wide** shocks (~RHIC): **full stopping**





- **Narrow** shocks (~LHC): **transparency**



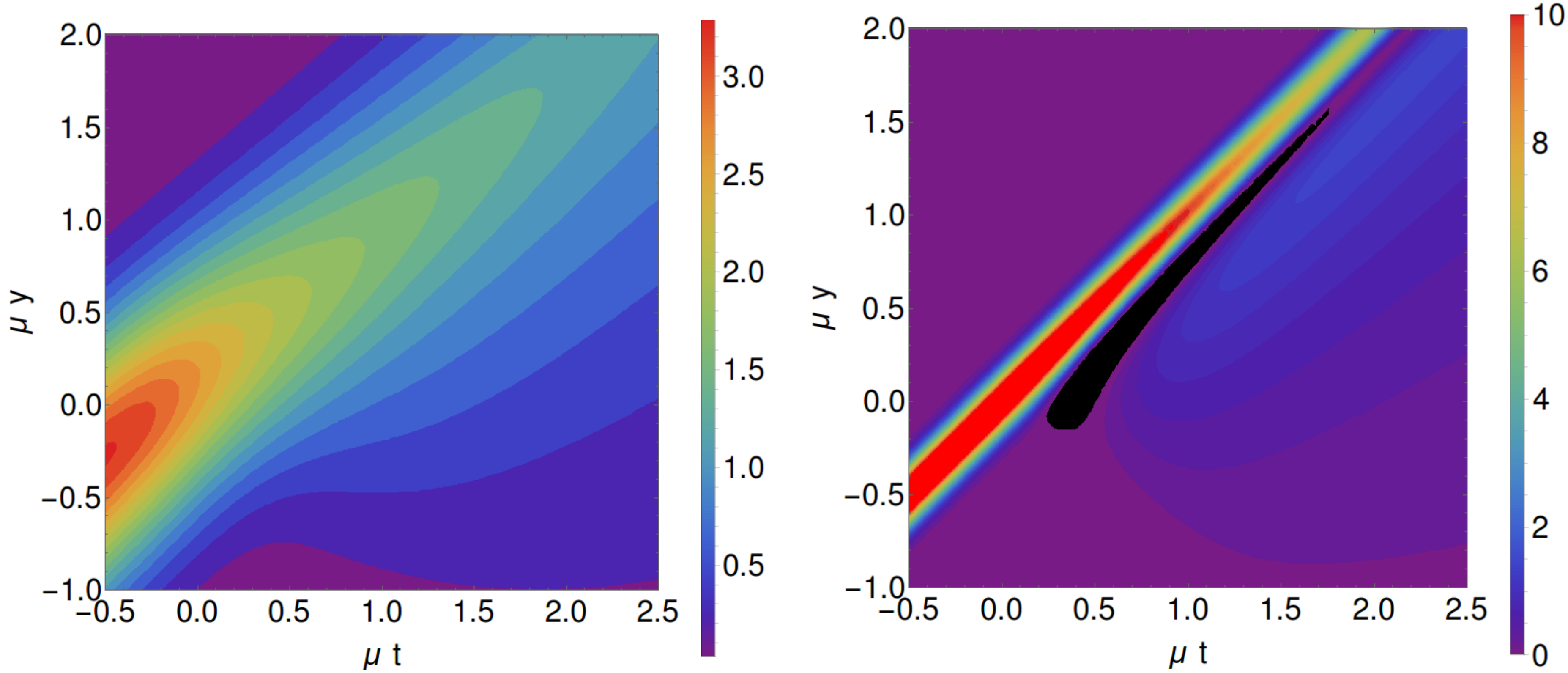


Violation of the null energy condition

"Well behaved" **classical theories satisfy** the **null energy condition** (NEC)

$$T_{\mu\nu}k^\mu k^\nu \geq 0\,, \quad k_\mu k^\mu = 0\,.$$

- In **quantum theories** the NEC **can be violated**. [Epstein 65]
- In **narrow shock wave collisions** the **null energy condition** (NEC) is **violated** in some region in the forward light cone **shortly after the collision**.

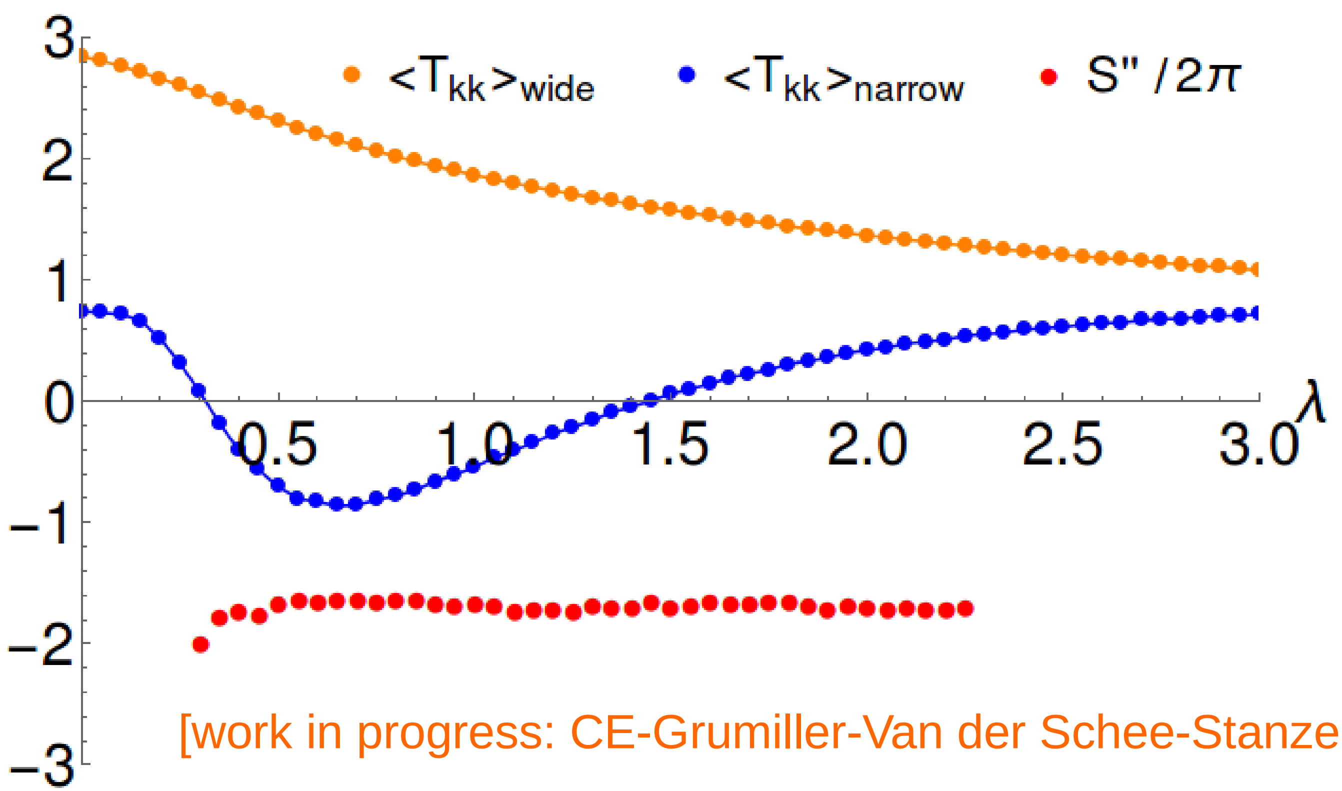


The quantum null energy condition is (preliminarily) fulfilled

Recently the **quantum null energy condition** (QNEC) was proposed [Bousso 15]

$$\langle T_{\mu\nu} k^\mu k^\nu \rangle \geq \frac{1}{2\pi} S'' \,.$$

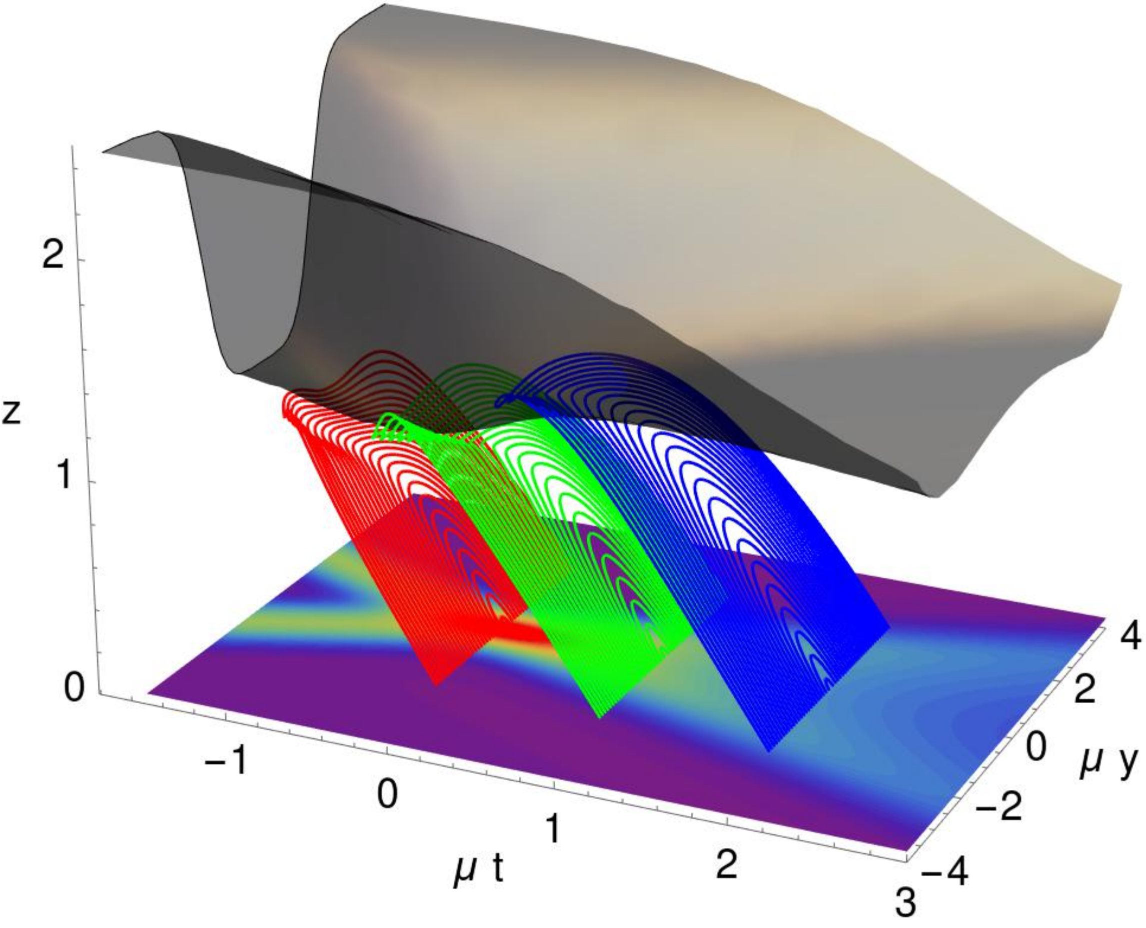
- Our **preliminary results** suggest that the **QNEC is fulfilled**.

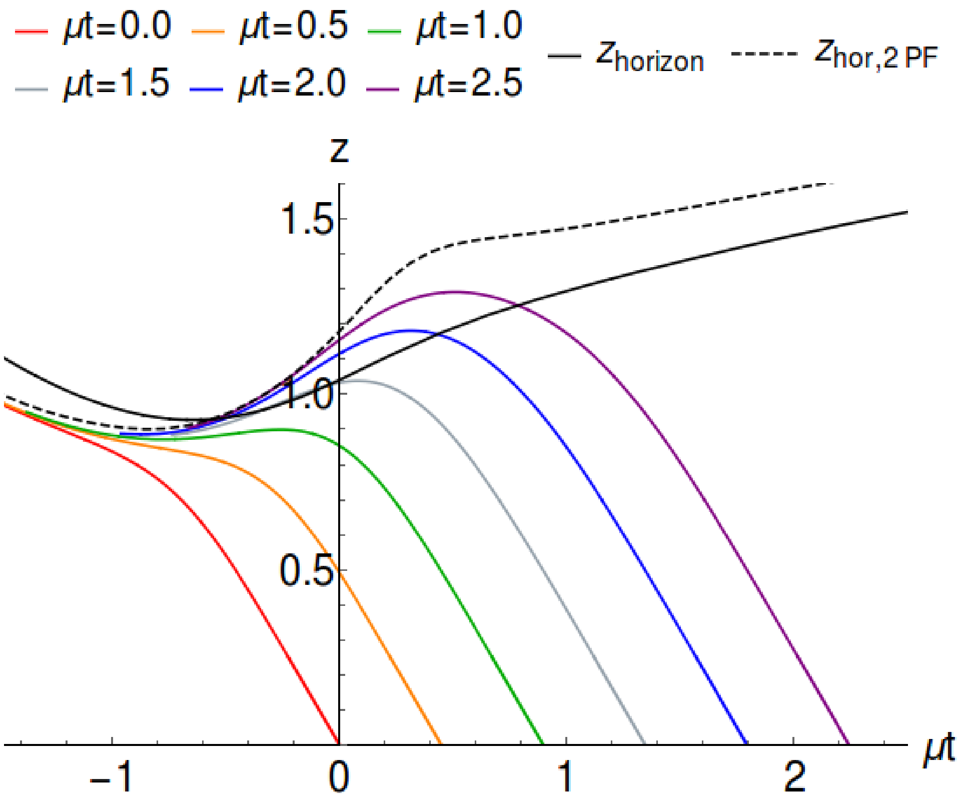


[work in progress: CE-Grumiller-Van der Schee-Stanzer]

Geodesics (2PF case)

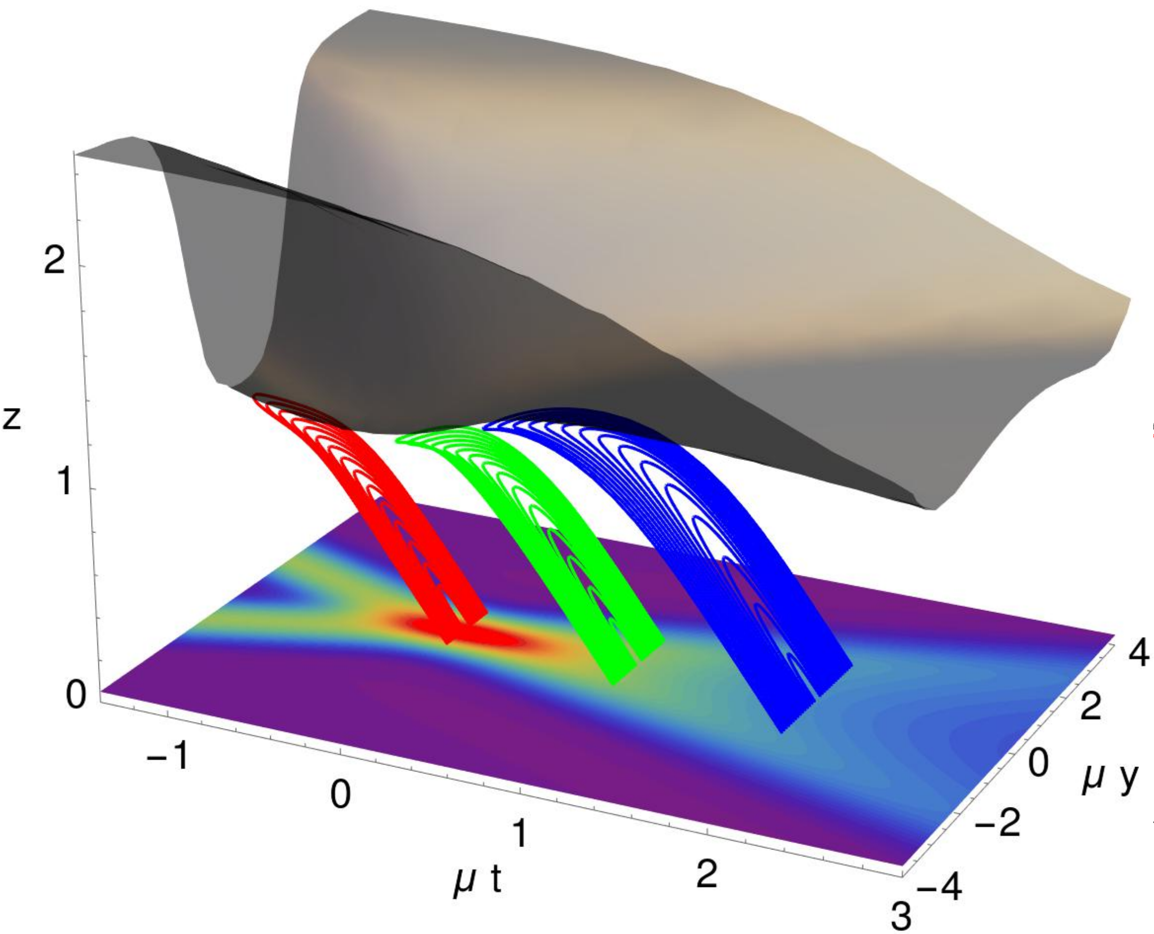
- In the case of **2-point functions** the **geodesics** can **reach beyond the apparent horizon**.
- Our numerical results suggest that there exists **maybe** something like a **2-point function horizon**.

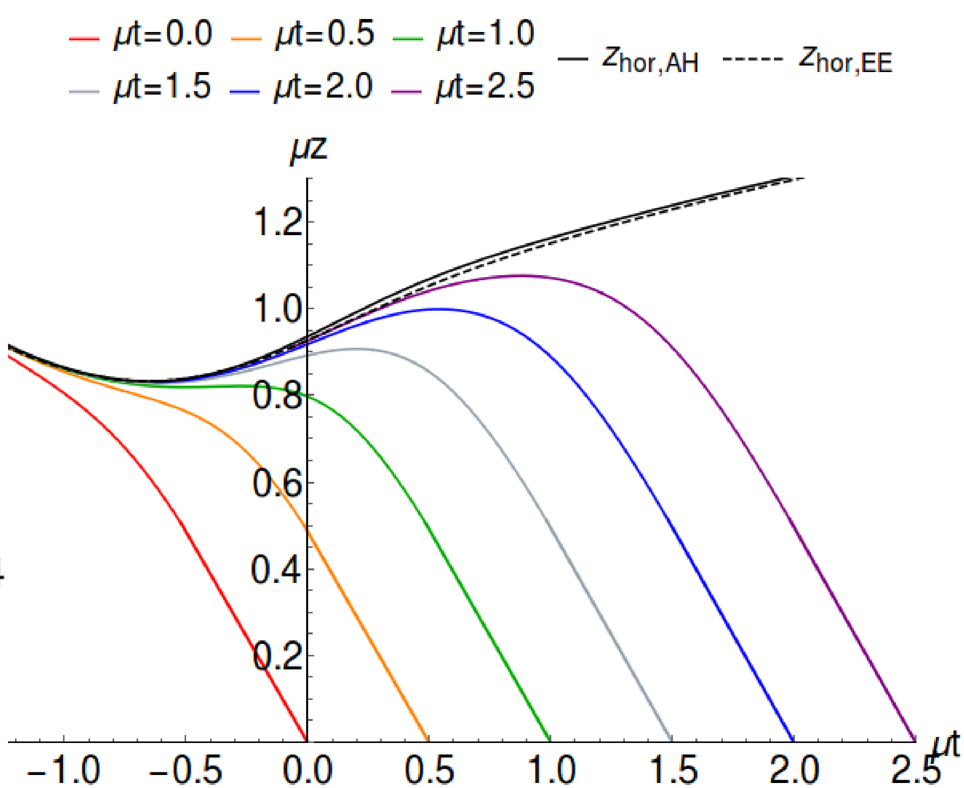




Extremal surfaces (EE case)

- In the case of **entanglement entropy** the **extremal surfaces do not reach beyond the AH**.
- If there is something like an **entanglement entropy horizon** it is **very close to the AH**.

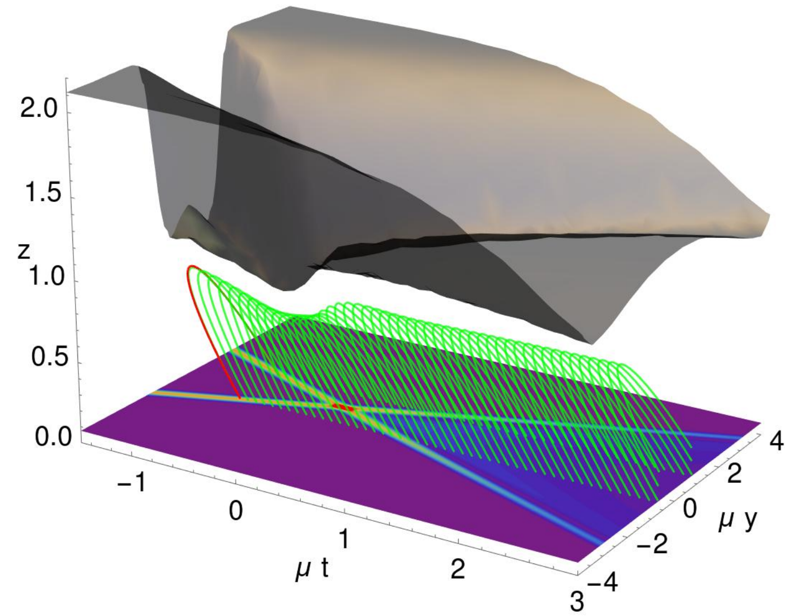


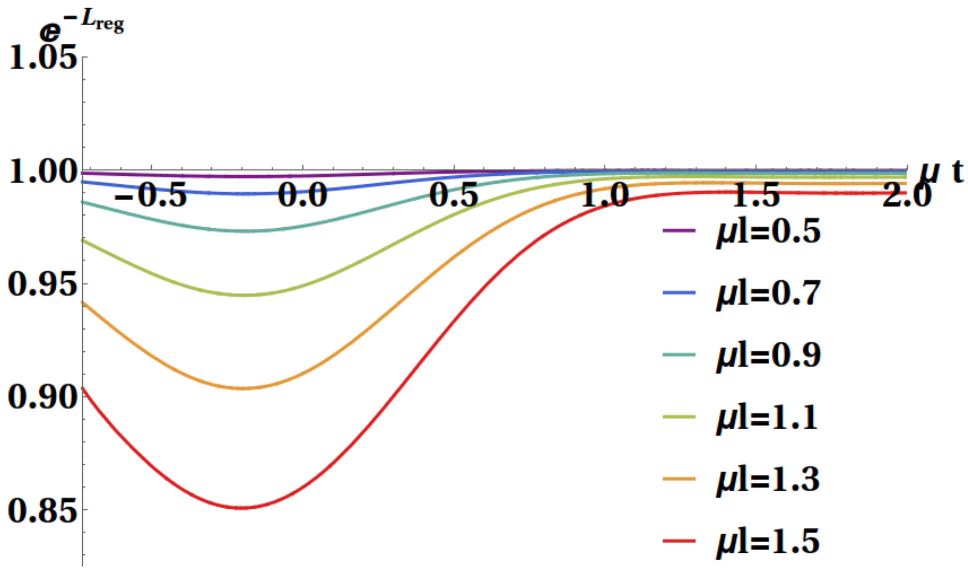


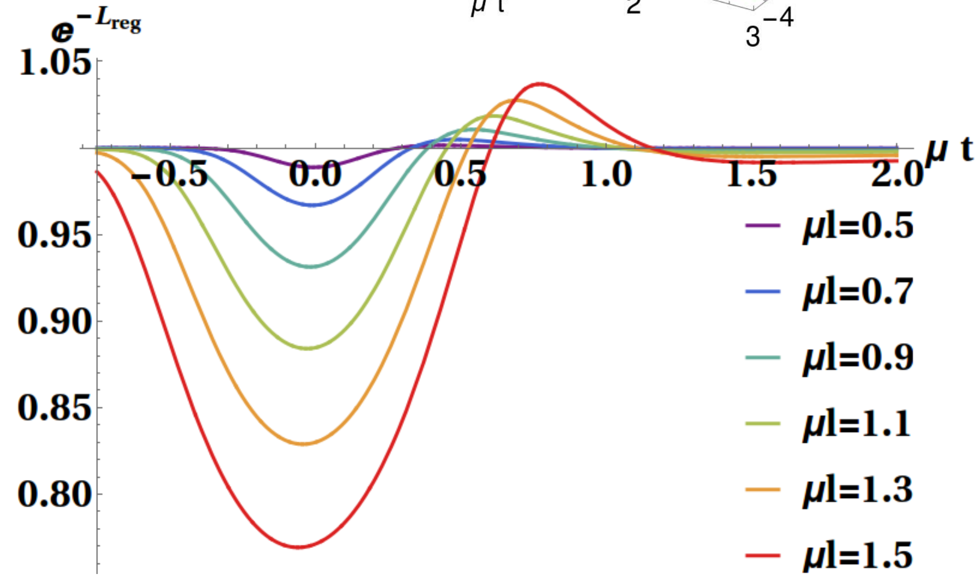
Time evolution of two-point functions

Characteristic behavior:

- **Rapid onset of linear de-correlation** before the collision.
- **Linear correlation restoration** right after the collision.
- **Correlation overshooting** of **narrow shocks**.



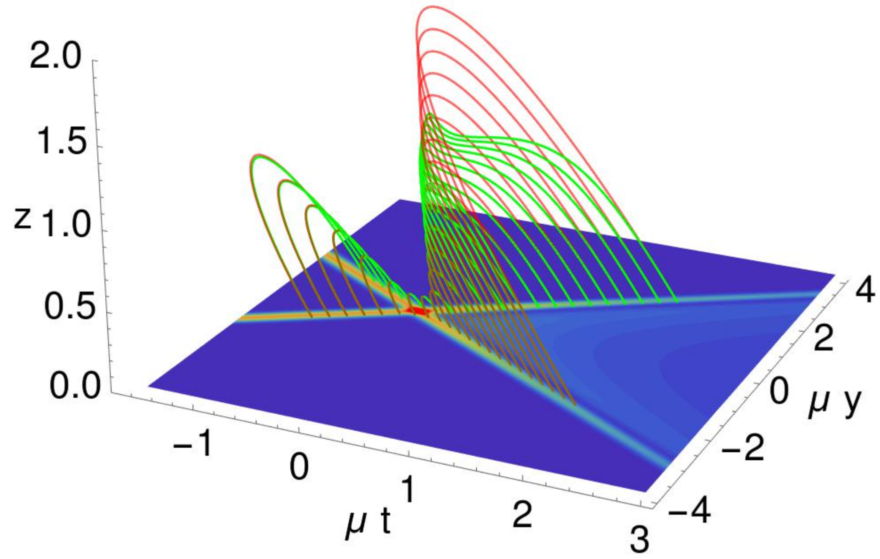


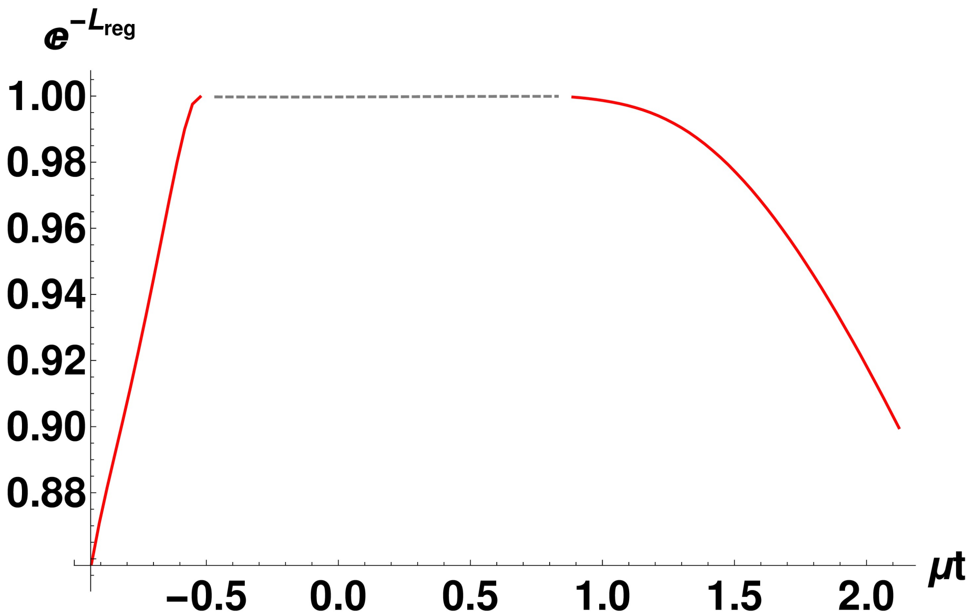


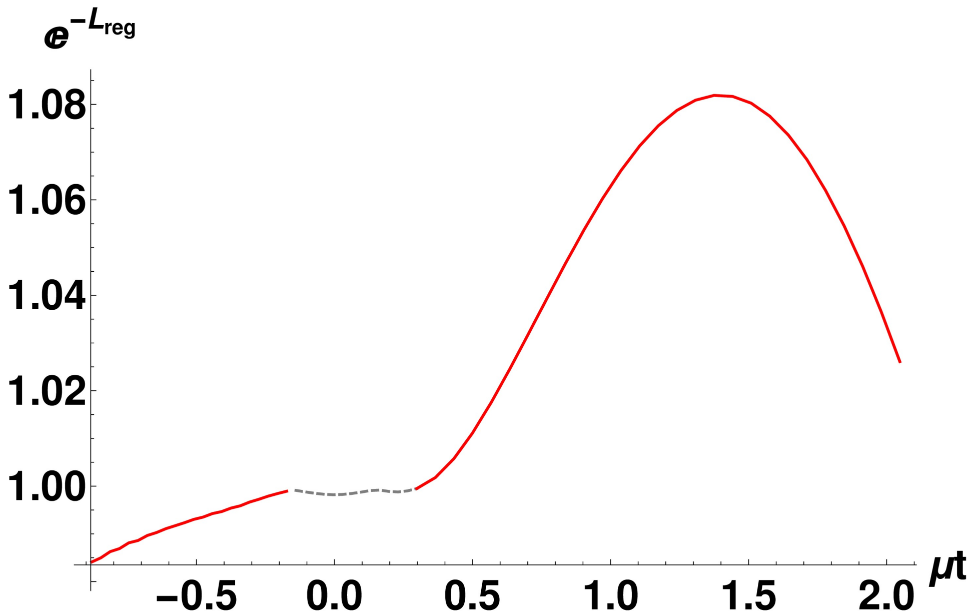
Correlations between shocks

Characteristic behavior:

- **Linear growth** before the collision.
- **Wide: Immediate decay** after the collision.
- **Narrow: Significant growth** of correlations after the collision before they eventually to decay.



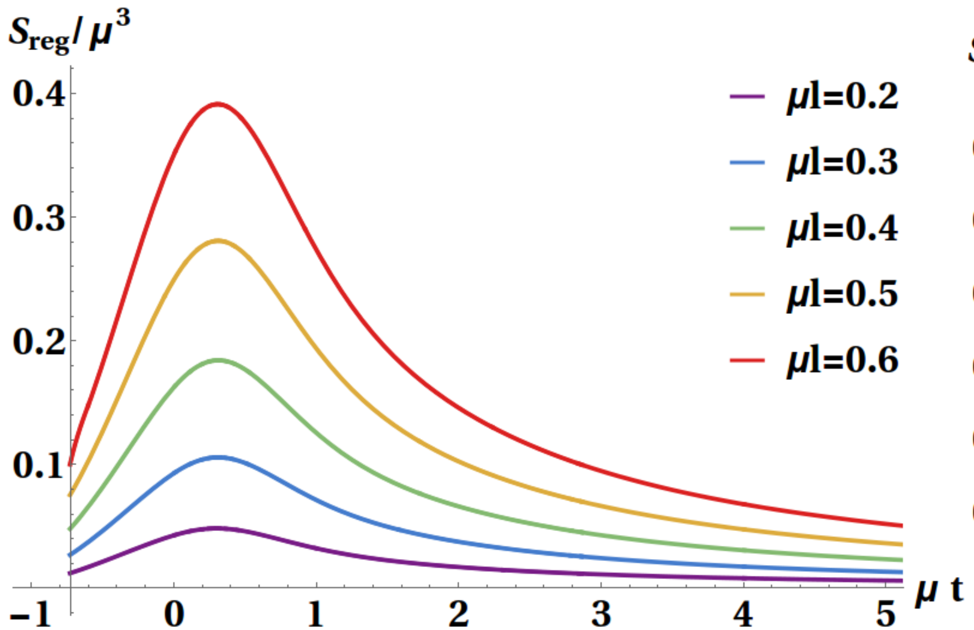


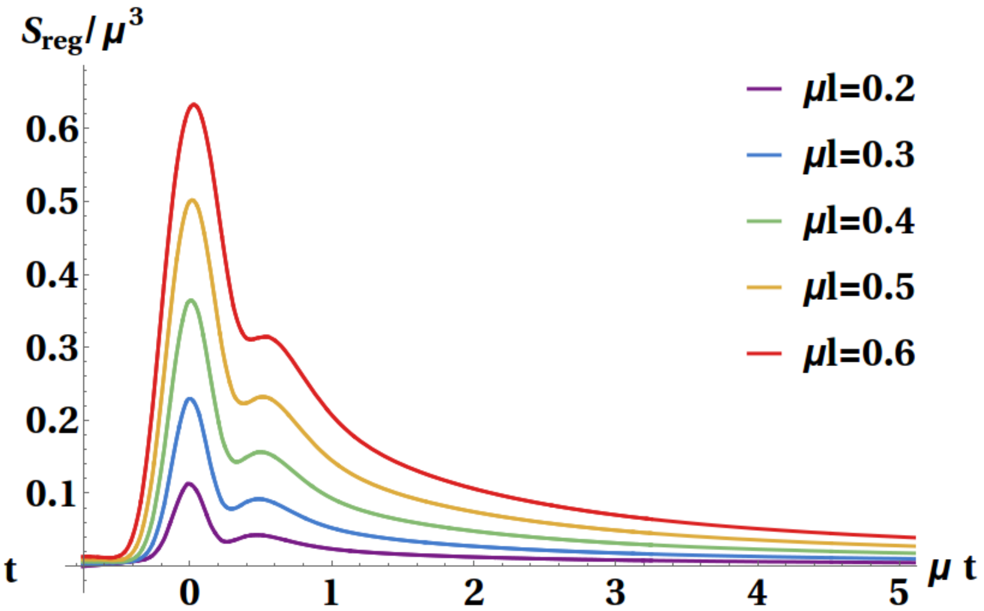


Time evolution of entanglement entropy

Characteristic behavior:

- **Rapid initial growth** when the shocks enter the entangling region.
- **Linear growth** when the shocks start to overlap.
- **Post collisional regime** which is a **featureless fall off for wide shocks** and shows a **characteristic shoulder for narrow shocks**.
- **Late time regime:** polynomial fall off very close to 1/t.





Summary

- **AdS/CFT** allows to study the **real time dynamics** of **strongly coupled QFT's** by solving the IVP of (classical) **supergravity** theories.
- The **NEC can be violated** in holographic shock wave collisions, but the **QNEC seems to be fulfilled**.
- **Entanglement entropy** may serve as an **order parameter** for the **full stopping–transparency transition**.
- Interestingly the **2-point functions contain information from behind the apparent** horizon, where the **entanglement entropy does not**.

Work in progress

- Improve the quality of the QNEC simulation.
  [CE-Grumiller-Van der Schee-Stanzer]
- **Going beyond supergravity**: string corrections, semi-holography, …
  [CE-Mukhopadhyay-Preiss-Rebhan-Stricker]