Entanglement Entropy in Shock Wave Collisions

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1. Introduction

- <u>Quark-Gluon Plasma in Heavy Ion Collisions</u>
  - **Heavy ion collision** experiments at RHIC and LHC produce a deconfied state of quarks and gluons, the so-called **quark-gluon plasma** (QGP).
  - The QGP in these experiments behaves like a **strongly coupled liquid**, not like a weakly coupled gas.
  - The plasma **thermalizes on a very short time scale** (≈10^{-23} sec), which is theoretically not well understood yet.
  - Due to the strong coupling **perturbative QCD is not suitable** to study the **quantum dynamics** of these collisions.
- <u>AdS/CFT Correspondence</u> [1]
  - AdS/CFT maps strongly coupled supersymmetric Yang-Mills (SYM) theory in 4D to classical gravity on 5D anti-de Sitter (AdS5) space.
  - We use the plasma in SYM theory as a **toymodel** for the experimentally realized QGP.
  - **Thermalization** in the strongly coupled 4D SYM theory is mapped to **black hole formation** in AdS5.

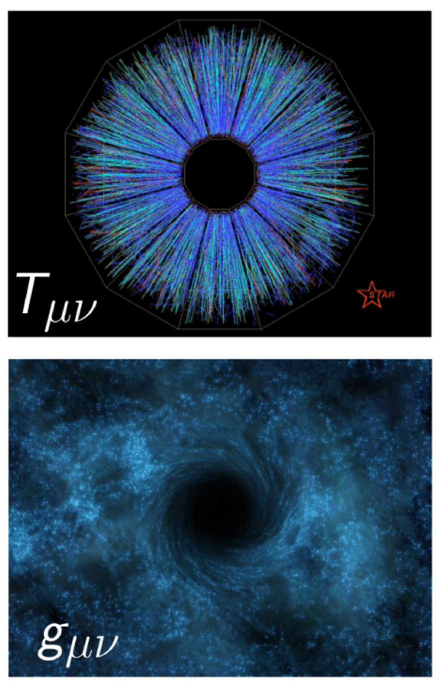

Figure 1: *AdS/CFT maps thermalization in the 4D gauge theory (top) to black hole formation in a 5D gravity theory (bottom).*

2. Local and Non-Local Observables from AdS/CFT

AdS/CFT allows to compute **expectation values of observables** in 4D SYM theory **from purely geometric objects** in the 5D gravity theory such as the **metric**, **geodesics** and **minimal surfaces**.

- <u>Local Observables</u>
  - The **energy-momentum tensor** can be extracted from the **metric** $g_{\mu\nu}$ near the boundary.

    $\langle T^{\mu\nu}(x)\rangle = -\frac{2}{\det g}\frac{\delta S}{\delta g^{\mu\nu}(x)}$
- <u>Non-Local Observables</u>
  - **Two-point functions** of gauge invariant operators $\mathcal{O}$ with large conformal weight Δ are given by the **length of geodesics** γ. [2]

    $\langle \mathcal{O}(t,\vec{x})\mathcal{O}(t,\vec{x}')\rangle \approx e^{-\Delta \mathrm{Length}(\gamma)}$
  - The **entanglement entropy** of a spatial region A is given by the **area of a minimal surface** Σ. [3]

    $S_A = -\mathrm{Tr}_A \rho_A \log \rho_A = \frac{\mathrm{Area}(\Sigma)}{4G_N}$

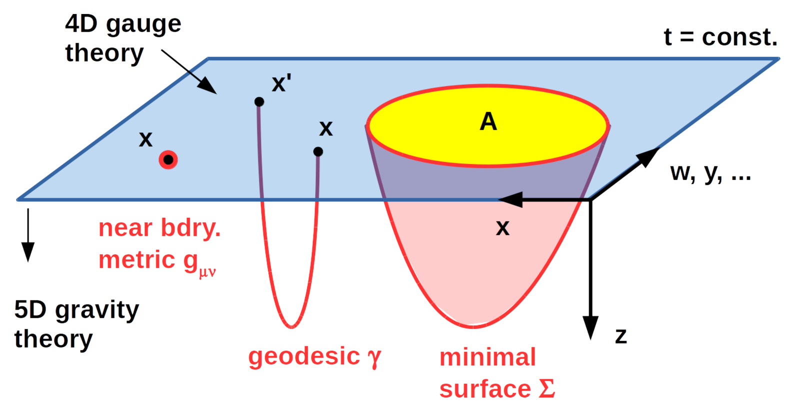

Figure 2: *Geometric description of the energy momentum tensor, two-point functions and entanglement entropy in terms of the near boundary metric, geodesics and minimal surfaces.*

3. Shock Wave Collisions in SYM Theory

Two **Lorentz contracted "nuclei"** are modelled as **Gaussian energy distributions** in SYM heading towards each other at the **speed of light**. The **time evolution** of the energy-momentum tensor is extracted from a **numerical relativity simulation** of **colliding gravitational shock waves** in the 5D gravity theory. [4]

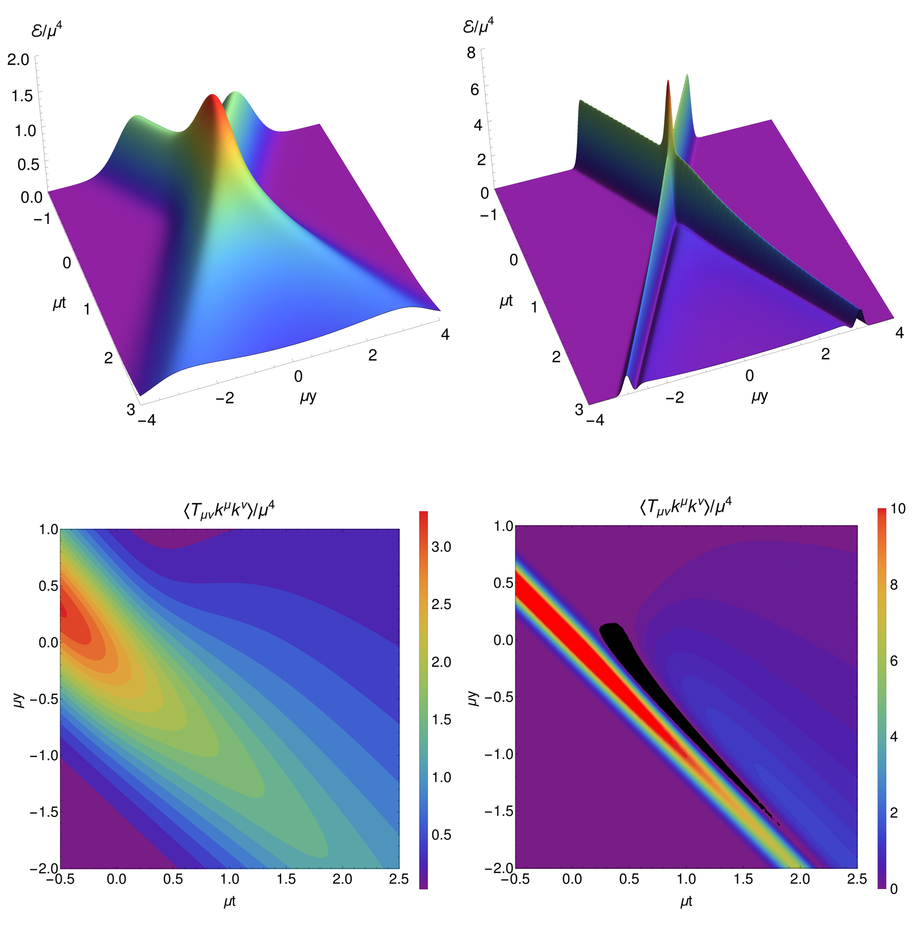

Figure 3: *Energy density $\mathcal{E}$ (top) and NEC (bottom) for wide (left) and narrow shocks (right). The region where the narrow shocks violate the NEC is shown in black (bottom right).*

- <u>Energy-Momentum Tensor</u>

$$\langle T^{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\begin{pmatrix} \mathcal{E} & \mathcal{S} & 0 & 0 \\ \mathcal{S} & \mathcal{P}_\parallel & 0 & 0 \\ 0 & 0 & \mathcal{P}_\perp & 0 \\ 0 & 0 & 0 & \mathcal{P}_\perp \end{pmatrix}$$

  - **Wide and narrow shocks** show **qualitatively different** behavior. [5]
- <u>Wide Shocks: Full-Stopping</u>
  - Wide shocks **stop each other** in the collision before they **explode hydrodynamically**.
  - Outgoing shocks are **slowed down**, **energy** and **pressure** are **positive**.
- <u>Narrow Shocks: Transparency</u>
  - Narrow shocks **pass each other** almost **"transparently"** without loosing velocity.
  - **Energy** and **pressure** can be **negative**.
- <u>Null Energy Condition (NEC)</u>

  $\langle k^\mu k^\nu T_{\mu\nu}\rangle \geq 0 \quad \forall \quad k^2 = 0$
  - **Narrow shocks** can violate the **NEC**. [6]
  - The **quantum null energy condition (QNEC)** is conjectured to give an **upper bound** for this violation. [8]

4. Geodesics in the Shock Wave Geometry

In the calculation of **two-point functions** and also for the **entanglement entropy** we need the **length of spacelike geodesics** that are **attached to the boundary** at z=0 and extend into the 5D shock wave geometry. These geodesics can be found by **numerically solving** the **geodesic equation** (1) subject to **boundary conditions** that fix the endpoints at the boundary ($z=0$) at some **spatial separation** l and time t. [7]

$$\ddot{X}^\mu + \Gamma^\mu_{\alpha\beta}\dot{X}^\alpha\dot{X}^\beta = -J\dot{X}^\mu, \quad s.t. \quad X^\mu|_{bdry} = (t,0,\pm l/2) \tag{1}$$

$$\mathrm{Length}(\gamma) = \int_\gamma \mathrm{d}\sigma\sqrt{g_{\mu\nu}X^\mu(\sigma)X^\nu(\sigma)} \tag{2}$$

- During the collision a **black hole horizon is formed** in the 5D shock wave geometry.
- **Near equilibrium** geodesics of small separation **do not to cross the horizon**.
- **Far from equilibrium** geodesics of large separation can **cross the horizon**.

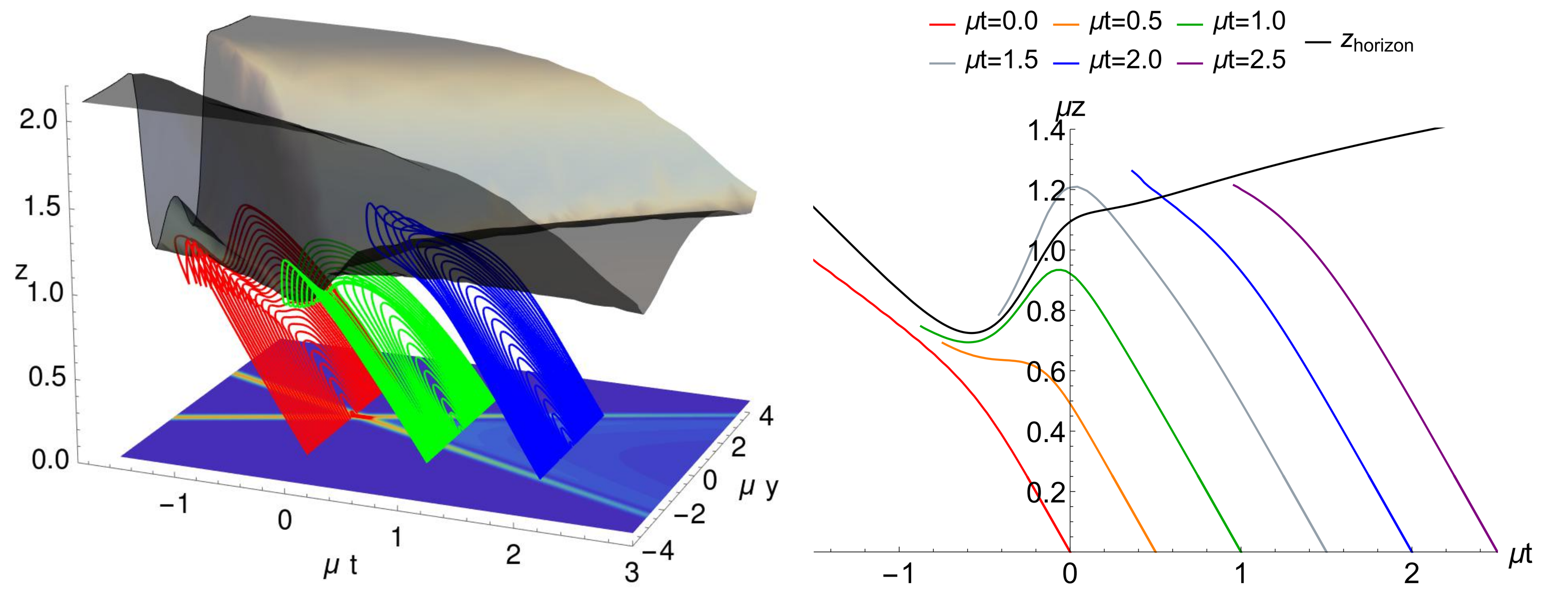

Figure 4: *Left: Black hole horizon (black), geodesics of different separation and time (red, green, blue), and energy contours at z=0. Right: Tip of the geodesics at different times.*

5. Two-Point Functions

<u>Time Evolution of Two-Point Functions</u>

- The **in-going shocks destroy** the **initial correlations** in the system.
- **During the collision**, when the shocks interact, **new correlations are formed** and the two-point function grows.
- The correlations of the **wide shocks** start to **grow clearly before the collision** ($t < 0$).
- For **narrow shocks** the correlations start to **grow close to the collision time** ($t = 0$) and significantly **overshoot their initial values** later on.

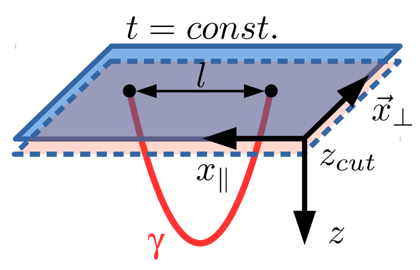

Figure 5: *We consider geodesics of finite sep. in the long. direction $x_\parallel$.*

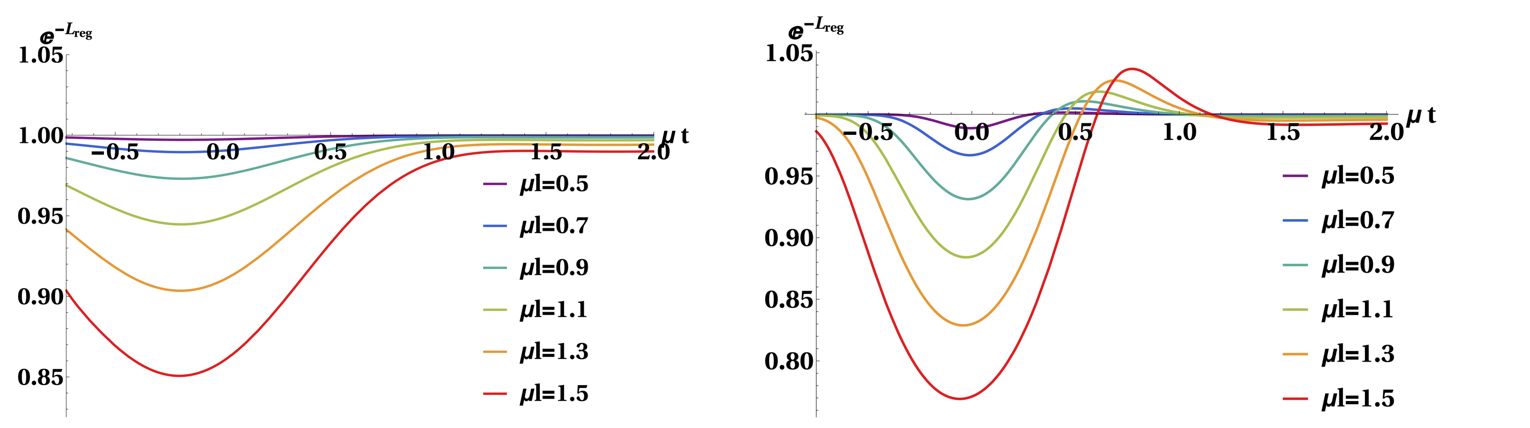

Figure 6: *Regularized two-point function of various separations l for wide shocks (left) and narrow shocks (right).*

6. Entanglement Entropy

<u>Time Evolution of Entanglement Entropy</u>

- As the **shocks enter the entangling region** the **entanglement entropy rapidly grows**.
- After the rapid initial growth follows a regime of **linear growth** which goes approx. **until the shocks collide**.
- **Narrow shocks** reach a **global maximum close to the collision time** ($t = 0$), for **wide shocks** the maximum is **clearly delayed**.
- For the **narrow shocks** there is an additional **local minimum after the collision**, which does not appear for the wide shocks.

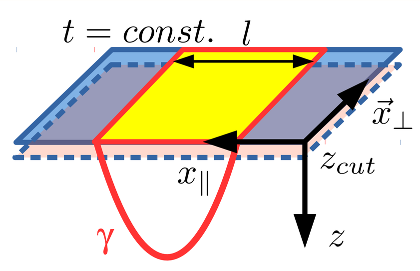

Figure 7: *We consider stripe regions of infinite extent in the trans. dir. $\vec{x}_\perp$. Minimal surfaces reduce to geodesics in an auxiliary spacetime.*

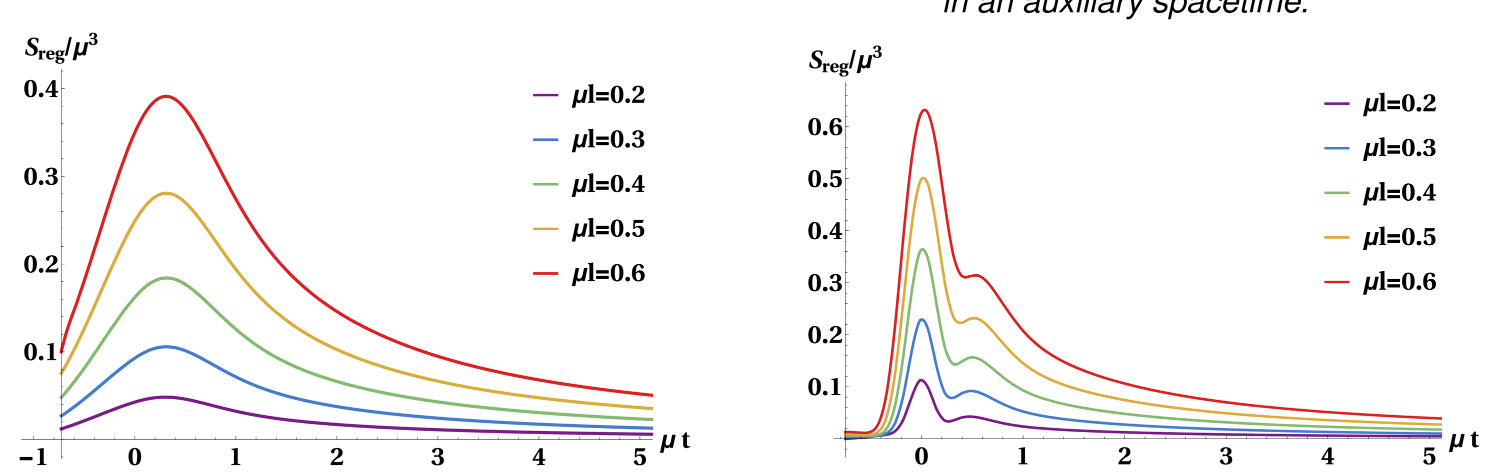

Figure 8: *Regularized entanglement entropy of various system sizes L for wide shocks (left) and narrow shocks (right).*

7. Summary

- We use collisions of **shock waves in SYM** theory as **toymodel for heavy ion collisions**.
- Within AdS/CFT **non-local observables** such as **two-point functions** and **entanglement entropy** can be computed from **geodesics** and **minimal surfaces** in the gravity theory.
- We study the **time evolution** of two-point functions and entanglement entropy and find **qualitatively different behavior for narrow and wide shocks**.
- **Narrow shocks** show **overshooting** in the **two-point function** and a **local minimum** in the **entanglement entropy after the collision**. These features do **not appear** in the **wide shocks**.

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9. References

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