Semi-Holography for Heavy Ion Collisions

Based on work with Ayan Mukhopadhyay, Florian Preis, Anton Rebhan, and Alexander Soloviev

Christian Ecker
(TU-Wien)

Utrecht
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TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

DOKTORATSKOLLEG PI
$\int dk\,\Pi$
Particles and Interactions

INSTITUTE FOR THEORETICAL PHYSICS

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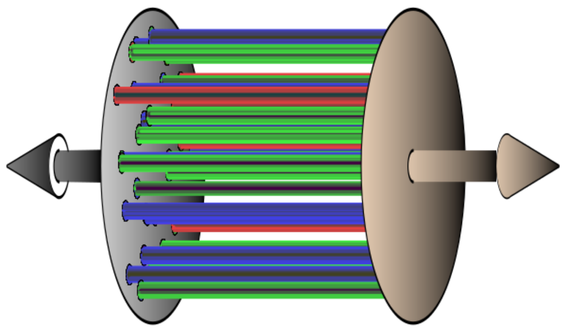
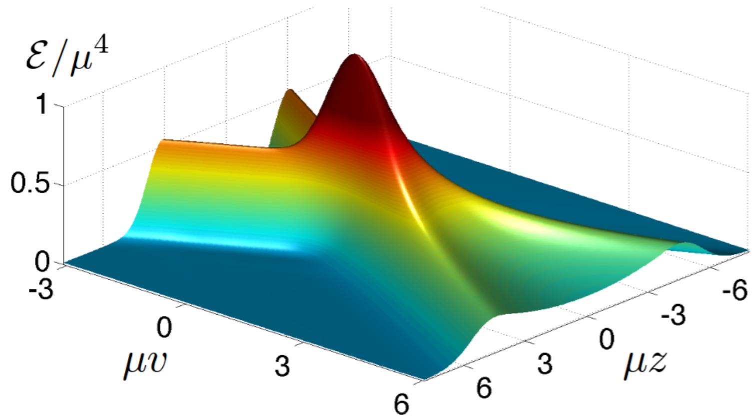
Introduction



Color Glass Condensate

- Early times ($\tau \lessapprox 0.1 fm/c$) semi-hard (Q_s) gluons dominate.
- Coupling is weak ($\alpha_s(Q_s) \ll 1$) but high occ. number $\sim 1/\alpha_s$.
- Effectively described by classical Yang-Mills fields (glasma).

[picture from: Gelis,Iancu,Jalilian-Marian and Venugopulan (1002.0333)]



Holographic Heavy Ion Collisions

- HICs from colliding grav. shock waves on AdS5.
- N=4 SYM theory at infinite coupling, not QCD.
- Fast hydrodynamization.
- Initial conditions?

[picture from: Chesler and Yaffe (1011.3562)]

Semi-holographic model realizes a self-consistent coupling between the Yang-Mills fields of the Color Glass Condensate and a strongly coupled AdS/CFT sector.

[proposed by: Iancu and Mukhopadhyay (1410.6448)
developed further by: Mukhopadhyay,Preis,Rebhan and Stricker (1512.06445)]

Combined Action

Combined action describing interaction between hard and soft modes

$$S = S_{YM} + W_{CFT}[g^{(b)}_{\mu\nu}, \phi^{(b)}, \chi^{(b)}]\,.$$

Classical Yang-Mills part describing the hard sector

$$S_{YM} = -\frac{1}{4N_c}\int d^4x\sqrt{-g^{YM}}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})\,.$$

The generating functional W_{CFT} describes a marginally deformed CFT in presence of the sources $(g^{(b)}_{\mu\nu}, \phi^{(b)}, \chi^{(b)})$, which are gauge invariant functions of the Yang-Mills fields.

$$g^{(b)}_{\mu\nu} = g^{YM}_{\mu\nu} + \frac{\boxed{\gamma}}{Q_s^4}t_{\mu\nu} \qquad t_{\mu\nu} = \frac{1}{N_c}\mathrm{Tr}(F_{\mu\alpha}F^{\alpha}_{\nu} - \tfrac{1}{4}g^{YM}_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}) \quad \text{(EMT)}$$

$$\phi^{(b)} = \frac{\boxed{\beta}}{Q_s^4}h \qquad h = \frac{1}{4N_c}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \quad \text{(Lagrangian density)}$$

$$\chi^{(b)} = \frac{\boxed{\alpha}}{Q_s^4}a \qquad a = \frac{1}{4N_c}\mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) \quad \text{(Pontryagin density)}$$

dimensionless couplings (α, β, γ) are free parameters of the model

Bulk Fields in the Holographic Sector

The sources $(g_{\mu\nu}^{(b)}, \phi^{(b)}, \chi^{(b)})$ couple to the energy momentum tensor ($\mathcal{T}_{\mu\nu}$), the glueball operator ($\mathcal{H}$) and the topological charge density operator ($\mathcal{A}$) of a holographic CFT.

The bulk fields dual to these operators are the metric (G_{MN}), the dilaton (ϕ) and the axion (χ) which have the following Fefferman-Graham expansion

$$G_{\mu\nu} = \frac{l^2}{z^2}\left(\underbrace{g_{\mu\nu}^{YM} + \frac{\gamma}{Q_s^4} t_{\mu\nu}}_{g_{\mu\nu}^{(b)}} + \ldots + z^4\left(\frac{4\pi G_5}{l^3}\boxed{\mathcal{T}_{\mu\nu}} + X_{\mu\nu}\right) + \mathcal{O}(z^4 \log z)\right)$$

(holographic EMT)

$$\phi = \underbrace{\frac{\beta}{Q_s^4} h}_{\phi^{(b)}} + \ldots + z^4 \frac{4\pi G_5}{l^3}\boxed{\mathcal{H}} + \mathcal{O}(z^6)$$

(Lagrangian density operator)

$$\chi = \underbrace{\frac{\alpha}{Q_s^4} a}_{\chi^{(b)}} + \ldots + z^4 \frac{4\pi G_5}{l^3}\boxed{\mathcal{A}} + \mathcal{O}(z^6)$$

(topological charge density operator)

Equations of Motion

The equations of motion follow from the variation of the combined action w.r.t. A_μ^a

$$\frac{\delta S}{\delta A_\mu} = \frac{\delta S_{YM}}{\delta A_\mu} + \int d^4x \left(\frac{\delta W_{CFT}}{\delta g^{(b)}_{\alpha\beta}} \frac{\delta g^{(b)}_{\alpha\beta}}{\delta A_\mu} + \frac{\delta W_{CFT}}{\delta \phi^{(b)}} \frac{\delta \phi^{(b)}}{\delta A_\mu} + \frac{\delta W_{CFT}}{\delta \chi^{(b)}} \frac{\delta \chi^{(b)}}{\delta A_\mu} \right) .$$

The standard Yang-Mills equations are modified by the marginal CFT-operators

$$\mathcal{T}^{\alpha\beta} = \frac{2}{\sqrt{-g^{(b)}}} \frac{\delta W_{CFT}}{\delta g^{(b)}_{\alpha\beta}} \qquad \mathcal{H} = \frac{1}{\sqrt{-g^{(b)}}} \frac{\delta W_{CFT}}{\delta \phi^{(b)}} \qquad \mathcal{A} = \frac{1}{\sqrt{-g^{(b)}}} \frac{\delta W_{CFT}}{\delta \chi^{(b)}}$$

$$\boxed{D_\mu F^{\mu\nu} = \frac{\gamma}{Q_s^4} D_\mu (\hat{\mathcal{T}}^{\mu\alpha} F^\nu_\alpha - \hat{\mathcal{T}}^{\nu\alpha} F^\mu_\alpha - \tfrac{1}{2} \hat{\mathcal{T}}^\alpha_\alpha F^{\mu\nu}) + \frac{\beta}{Q_s^4} D_\mu (\hat{\mathcal{H}} F^{\mu\nu}) + \frac{\alpha}{Q_s^4} (\partial_\mu \hat{\mathcal{A}}) \tilde{F}^{\mu\nu}}$$

where $\hat{\mathcal{T}}^{\mu\nu} = \frac{\sqrt{-g^{(b)}}}{\sqrt{-g^{YM}}} \mathcal{T}^{\mu\nu}, \ldots$ are tensors living in the background metric $g^{YM}_{\mu\nu}$.

Total Energy Momentum Tensor

The energy momentum tensor of the combined system contains contributions form the classical Yang-Mills sector, the holographic sector and mix-terms

$$\frac{2}{\sqrt{-g^{YM}}}\frac{\delta S}{\delta g^{YM}_{\mu\nu}} = T^{\mu\nu} = t^{\mu\nu} + \hat{\mathcal{T}}^{\mu\nu} + \mathrm{mixed}\,.$$

For $g^{YM}_{\mu\nu} = \eta_{\mu\nu}$ the mix-terms are given by

$$\mathrm{mixed} = -\frac{\gamma}{Q_s^4 N_c}\hat{\mathcal{T}}^{\alpha\beta}[\mathrm{Tr}(F^{\mu}_{\alpha}F^{\nu}_{\beta}) - \frac{1}{2}\eta_{\alpha\beta}\mathrm{Tr}(F^{\mu\rho}F^{\nu}_{\rho}) + \frac{1}{4}\delta^{\mu}_{(\alpha}\delta^{\nu}_{\beta)}\mathrm{Tr}(F^2)]$$
$$-\frac{\beta}{Q_s^4 N_c}\hat{\mathcal{H}}\mathrm{Tr}(F^{\mu\alpha}F^{\nu}_{\alpha}) - \frac{\alpha}{Q_s^4}\eta^{\mu\nu}\hat{\mathcal{A}}a\,.$$

The total EMT is conserved on flat space, the holographic EMT fulfills a Ward identity

$$\boxed{\partial_\mu T^{\mu\nu} = 0} \qquad \nabla_\mu \mathcal{T}^{\mu\nu} = -\frac{\beta}{Q_s^4}\mathcal{H}\nabla^\nu h \quad \text{with } \nabla_\mu \text{ Levi-Civita of } g^{(b)}_{\mu\nu}$$

[for proof see: Mukhopadhyay,Preis,Rebhan,Stricker (1512.06445)]

Self-Consistency Loop

Generate initial guess by solving sourceless Yang-Mills equations

$$D_\mu F^{\mu\nu} = 0$$

A^μ_{ini}

Compute sources for gravity system

$$g^{(b)}_{\mu\nu} = g^{YM}_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu}[A]$$

$$\phi^{(b)} = \frac{\beta}{Q_s^4} h[A] \quad \chi^{(b)} = \frac{\alpha}{Q_s^4} a[A]$$

$\{g^{(b)}_{\mu\nu}, \phi^{(b)}, \chi^{(b)}\}$

Solve the gravity problem with boundary sources and extract normalizable modes

$$G_{\mu\nu} = \ldots + z^4 \frac{4\pi G_5}{l^3} \mathcal{T}_{\mu\nu} + \ldots$$

$$\phi = \ldots + z^4 \frac{4\pi G_5}{l^3} \mathcal{H} + \ldots$$

$$\chi = \ldots + z^4 \frac{4\pi G_5}{l^3} \mathcal{A} + \ldots$$

$\{\mathcal{T}^{\mu\nu}, \mathcal{H}, \mathcal{A}\}$

Solve the modified Yang-Mills equation

$$D_\mu F^{\mu\nu} = J^\nu[\mathcal{T}, \mathcal{H}, \mathcal{A}]$$

A^μ_{new}

Check Ward identity of hol. EMT

$$\nabla_\mu \mathcal{T}^{\mu\nu} = -\frac{\beta}{Q_s^4} \mathcal{H} \nabla^\nu h$$

and **conservation of total EMT**

$$\partial_\mu T^{\mu\nu} = 0$$

True

Finish

False

A^μ_{new}

Example

Consider homogeneous and isotropic SU(2) Yang-Mills theory on 2+1 dimensional Minkowski space ($g^{YM}_{\mu\nu} = \eta_{\mu\nu}$) with dilaton coupling only $(\alpha = \gamma = 0, \beta \neq 0)$.

In this case the Yang-Mills equations for the combined system reduce to

$$D^\mu F^a_{\mu\nu} = \frac{\beta}{Q_s^4} D_\mu(\hat{\mathcal{H}} F_{\mu\nu}) \quad \text{with} \quad D_\mu = \partial_\mu - i\lambda A^a_\mu T^a, \quad T^a = \tfrac{1}{2}\sigma^a_{Pauli}\,.$$

Using temporal gauge ($A^a_0 = 0$) and locking of color and spacial indices ($A^a_i(t) \propto \delta^a_i$) (plus isotropy of the EMT) reduces the gauge field to a single function of time

$$A^a_\mu(t) = f(t)\delta^a_\mu\,.$$

For this choice the Yang-Mills equations simplify to a single non-linear ODE

$$(1-\beta\mathcal{H})f'' - \beta\mathcal{H}'f' + \lambda^2(1-\beta\mathcal{H})f^3 = 0\,.$$

The source for the dilaton in the holography sector is given by

$$\phi^{(b)} = \frac{\beta}{Q_s^4} h \qquad \text{with} \qquad h = \frac{1}{2}(\lambda^2 f^4 - 2f'^2)\,.$$

Holographic Sector

Homogeneous and isotropic AdS4 black brane coupled to a massless scalar field

$$ds^2 = -A(z,t)dt^2 - \frac{2dtdz}{z^2} + S^2(z,t)\big(dx_1^2 + dx_2^2\big) \qquad \phi = \phi(z,t)$$

The Einstein-Klein-Gordon system needs to be solved numerically

$$R_{ab} - \tfrac{1}{2}g_{ab}R + \Lambda g_{ab} = \tfrac{1}{2}\partial_a\phi\partial_b\phi - \tfrac{1}{4}g_{ab}\partial^c\phi\partial_c\phi \qquad \Box_g\phi = 0$$

subject to boundary conditions coming from the classical Yang-Mills field

$$A = \tfrac{1}{z^2} - \tfrac{3}{4}(\phi^{(b)\prime})^2 - \mathcal{T}^{00}z + \mathcal{O}(z^2) \qquad S = \tfrac{1}{z} - \tfrac{1}{8}(\phi^{(b)\prime})^2 z + \mathcal{O}(z^3)$$

$$\phi = \phi^{(b)} + z\phi^{(b)\prime} + z^2\left(\tfrac{1}{3}\mathcal{H} + \tfrac{1}{4}(\phi^{(b)\prime})^3 - \tfrac{1}{3}\phi^{(b)\prime\prime\prime}\right) + \mathcal{O}(z^4)$$

The solution has to satisfy the Ward identity

$$\mathcal{T}_0^{0\prime} = \mathcal{H}\phi^{(b)\prime}\,.$$

Initial Conditions

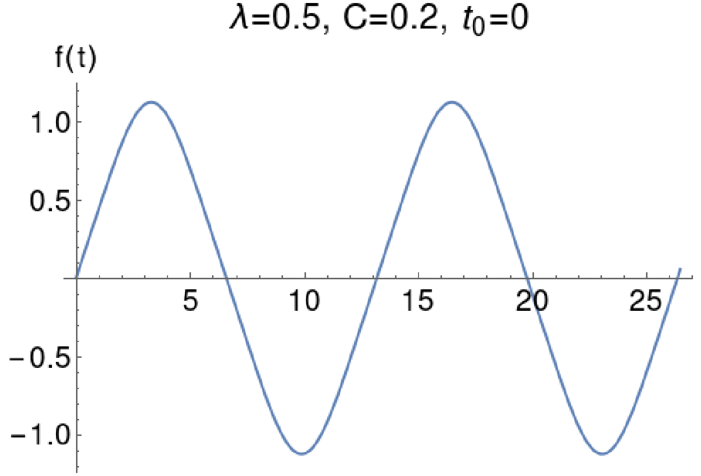
The iterative procedure is initialized with a solution of the non-coupled ($\beta = 0$) Yang-Mills equation which is given by a Jacobi elliptic function.

$$f'' + \lambda^2 f^3 = 0$$

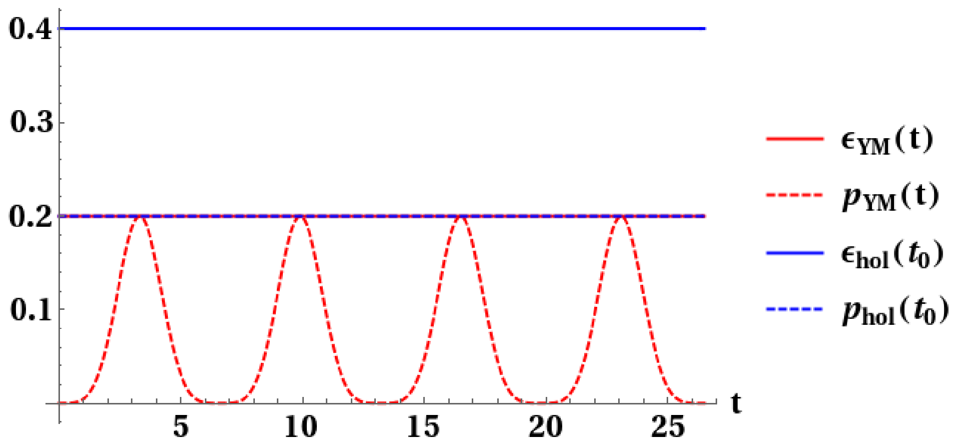
$$f(t) = \pm \sqrt[4]{\frac{2C}{\lambda^2}} \mathrm{sn}(\sqrt[4]{\frac{C\lambda^2}{2}} |t - t_0|| - 1)$$

$$\epsilon_{YM} = \frac{1}{2}\lambda^2 f^4 + (f')^2 \qquad p_{YM} = \frac{1}{2}\lambda^2 f^4$$

Solution has constant energy: $\epsilon_{YM} = C$



On the holography side we have to set the initial energy $\mathcal{T}^{00}(t=0)$ and the radial profile of the scalar field $\phi(z, t=0)$ on the initial time-slice in the bulk.



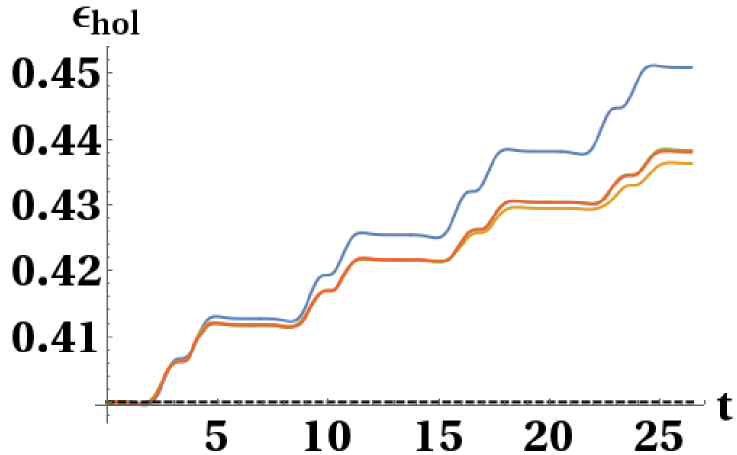
All time-derivatives of the non-coupled solution vanish at t=0.

Ward identity fixes $\mathcal{H}(t=0) = 0$.

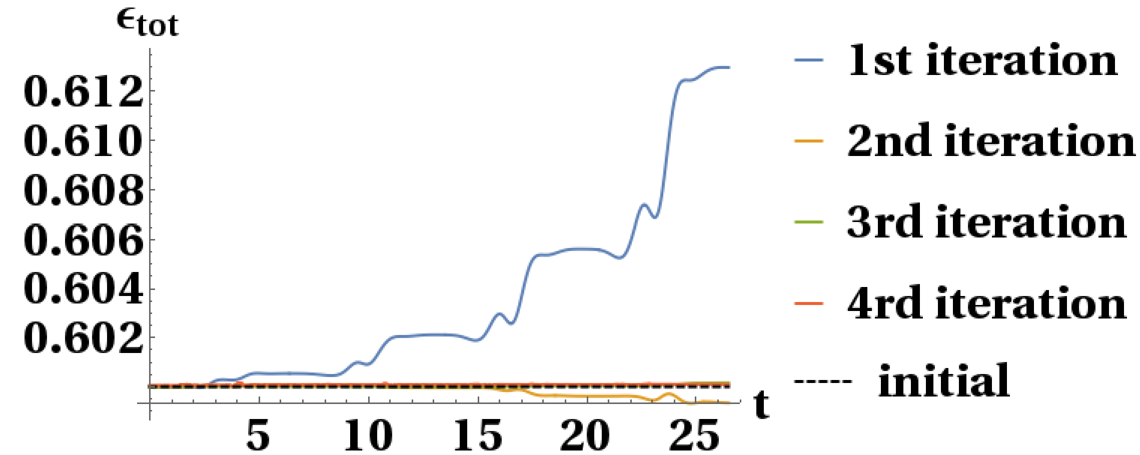
The initial radial profile is constant

$$\phi(z, t=0) = \phi^{(b)}(0) = -\beta C$$

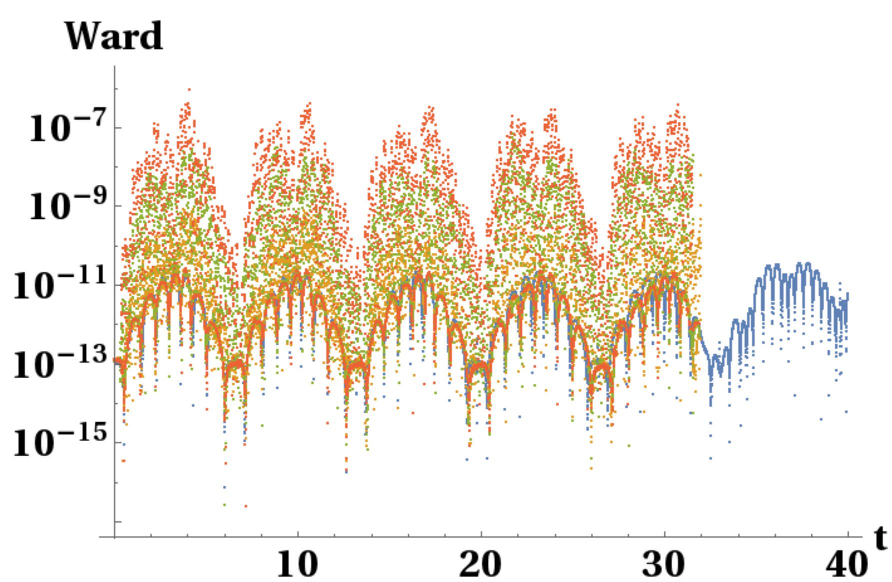
Iterations



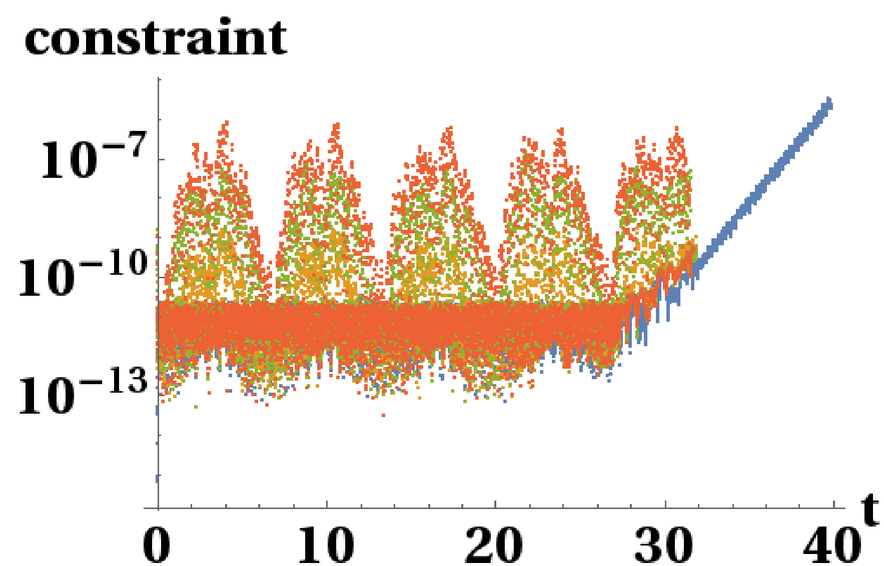
ϵ_{hol}
0.45
0.44
0.43
0.42
0.41
5
10
15
20
25
t



ϵ_{tot}
0.612
0.610
0.608
0.606
0.604
0.602
5
10
15
20
25
t
1st iteration
2nd iteration
3rd iteration
4rd iteration
initial

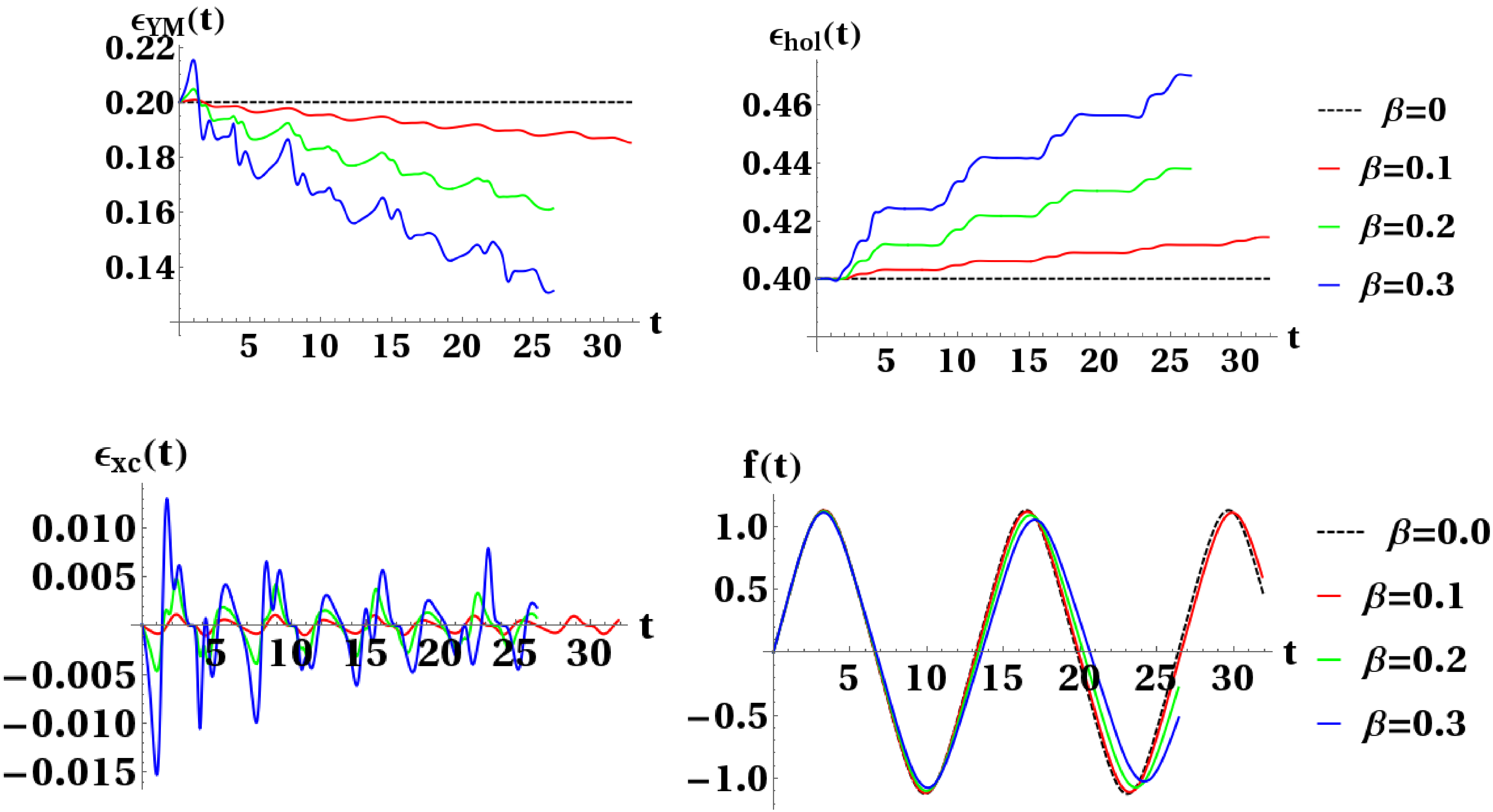


Ward
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10
20
30
40
t



constraint
10^{-7}
10^{-10}
10^{-13}
0
10
20
30
40
t

Results



$\epsilon_{YM}(t)$
0.22
0.20
0.18
0.16
0.14
5 10 15 20 25 30
t
$\epsilon_{hol}(t)$
0.46
0.44
0.42
0.40
5 10 15 20 25 30
t
β=0
β=0.1
β=0.2
β=0.3
$\epsilon_{xc}(t)$
0.010
0.005
−0.005
−0.010
−0.015
5 10 15 20 25 30
t
f(t)
1.0
0.5
−0.5
−1.0
5 10 15 20 25 30
t
β=0.0
β=0.1
β=0.2
β=0.3

Summary

First semi-holographic simulations featuring energy exchange between classical Yang-Mills sector and a holographic CFT.

Successful proof of principle: the semi-holographic model can be solved with an iterative procedure.

Energy is transfered from the weakly coupled (UV) to the strongly coupled (IR) sector.

Increasing the mutual coupling, irrespective of sign, increases the rate of energy transfer.

Ongoing work

Improve the numerics such that it is long time stable, allows for larger couplings and smaller initial energy in the holographic sector.

Simulations for 4 dimensional Yang-Mills coupled to AdS5/CFT4 (numerically harder).

Include tensor and axion coupling.

Ultimate goal: semi-holographic shock wave collisions with CGC initial conditions.