Numerical Holography
Numerical Relativity & AdS/CFT

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Plan of the talk

First part: Holographic Thermalization

- Pre-equilibrium dynamics in relativistic heavy ion collisions
- The AdS/CFT approach: thermalization = black hole formation
- Numerical relativity on AdS: the Chesler-Yaffe method
- Holographic toy models: homogeneous isotropization, shock waves, . . .

Second part: Holographic Entanglement Entropy

- Entanglement entropy
- The Ryu-Takayanagi proposal: entanglement entropy from extremal surfaces
- Geodesics on time dependent backgrounds: a glance behind the horizon?

Summary and Outlook

Relativistic Heavy-Ion Collisions

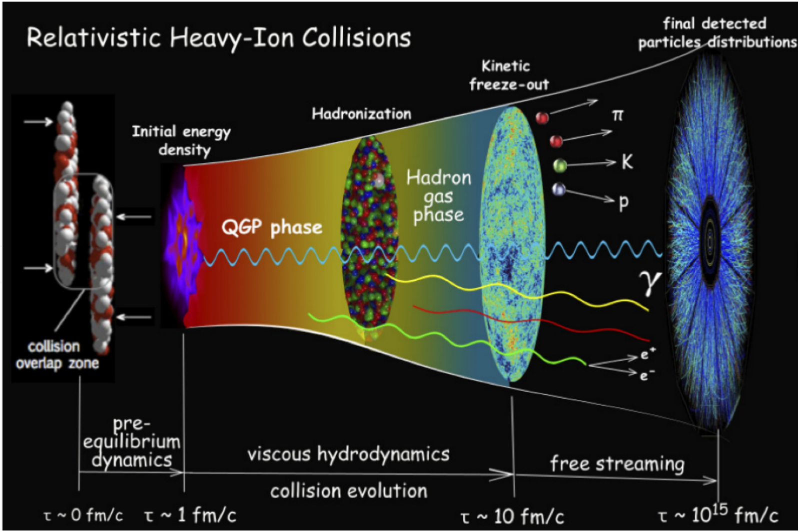


Fig. by P. Sorensen and C. Shen

Pre-equilibrium dynamics in HICs

thermalization (hydronization) = equilibration to hydrodynamic regime
After the thermalization time the EMT is well described by hydrodynamics.

In principle **we know the theory** which describes the pre-equilibrium phase: **QCD**

However we **can not solve** QCD in this phase:

- Perturbative QCD is not valid due to strong coupling.
- Time dependent processes are problematic for lattice QCD.

Alternative approach:

- Study the dynamics of a toy model for QCD: strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory
- Unfortunately we also can not (directly) solve $\mathcal{N} = 4$ SYM.
- However the AdS/CFT correspondence maps $\mathcal{N} = 4$ SYM to classical gravity.
- General relativity we can do very well!

Holographic principle and AdS/CFT correspondence

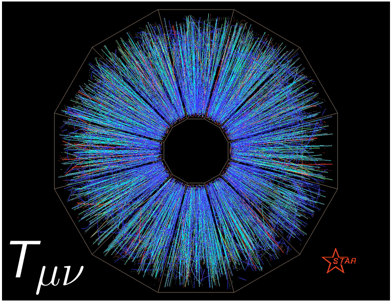
- **Holographic Principle** ['t Hooft 93, Susskind 94]:
  A theory of (quantum) gravity in n dimensions has an equivalent description in terms of a theory without gravity in $n-1$ dimensions.

- **AdS/CFT correspondence** [Maldacena 97]:
  $\mathcal{N}=4$ supersymmetric $SU(N_c)$ Yang-Mills theory (SYM) is equivalent to type IIB string theory on asympt. $AdS_5 \times S^5$.

- **We consider a certain limit of AdS/CFT**:
  Strongly coupled, large N_c $\mathcal{N}=4$ SYM theory is equivalent to classical gravity on AdS_5.

Holographic thermalization

thermalization = **black hole formation**



- AdS/CFT translates the physics of thermalization/equilibration on the field theory side to the formation of a black hole on AdS.
- Temperature and entropy of the black hole translate to temperature and entropy of the field theory.
- AdS/CFT relates the EMT $T_{\mu\nu}$ of the field theory to the metric $g_{\mu\nu}$ of the AdS black hole.

Numerical relativity on AdS: the Chesler-Yaffe method

The aim is to solve the gravitational initial value problem (+BC's) on AdS to get the metric $g_{\mu\nu}$.

Characteristic formulation:

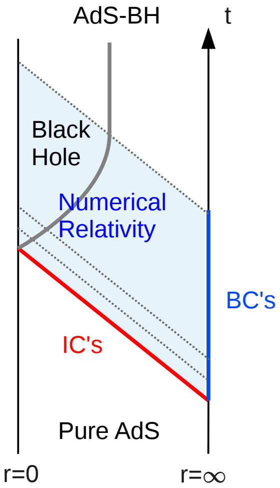
$$ds^2 = dt[-Adt + \beta dr + 2F_i dx^i] + \Sigma^2 h_{ij} dx^i dx^j$$

- This special parametrization of AdS decouples the Einstein eqs. into a nested set of linear ODEs.
- ODEs are solved with standard numerical techniques. (Chebychev spectral method, ...)

Out-of-equilibrium configurations:

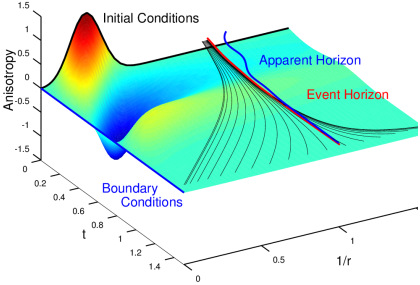
- **IC's**: anisotropy, shock waves, ...
- **BC's**: flat boundary, boundary with time dep. curvature, ...

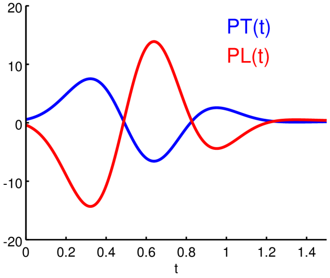
P. Chesler, L. Yaffe, 1309.1439



Homogeneous isotropization: the beginner problem

- Spatial homogeneous, rotational symmetric in transverse plane, but allows for time dependent pressure anisotropy.
- Line element: $ds^2 = 2drdt - A(r,t)dt^2 + \Sigma(r,t)^2\left(e^{-2B(r,t)}dx_\parallel^2 + e^{B(r,t)}d\vec{x}_\perp^2\right)$
- Energy momentum tensor: $\langle T_{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\mathrm{diag}[\epsilon, P_\parallel(t), P_\perp(t), P_\perp(t)]$

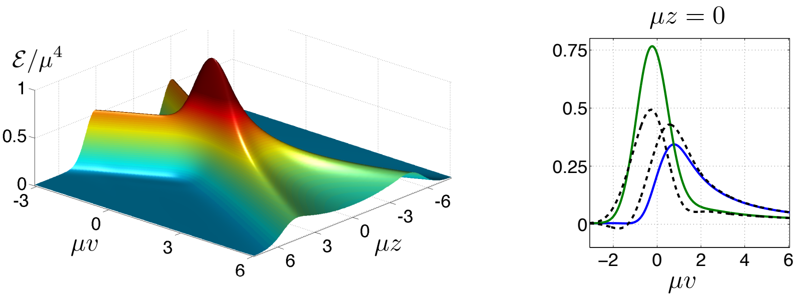




P. Chesler, L. Yaffe, 0812.2053

Including longitudinal dynamics: hom. shock waves

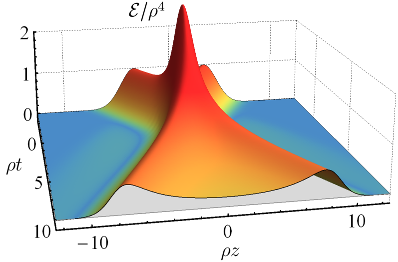
- Lorentz contracted ions are modeled as is homogeneous and infinitely extended energy distribution in the transverse plane with a Gaussian profile in the longitudinal direction.
- Gaussians move at the speed of light in the longitudinal direction.
- Hydrodynamics applies even when the initial Gauians are still in contact and the pressure anisotropy is large.

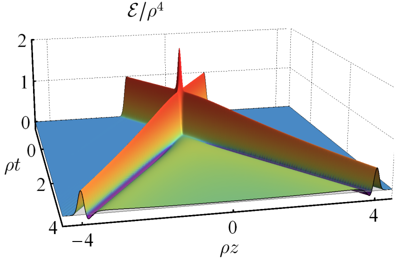


P. Chesler, L. Yaffe, 1011.3562

Homogeneous shock waves: dynamical cross over

- Dynamical cross over: wide and narrow shocks give qualitatively different results
- Wide shocks (full stopping): Au Ions at RHIC, Lorentz contraction ≈ 100.
- Narrow shocks (transparent): Pb Ions at LHC, Lorentz contraction ≈ 1000.

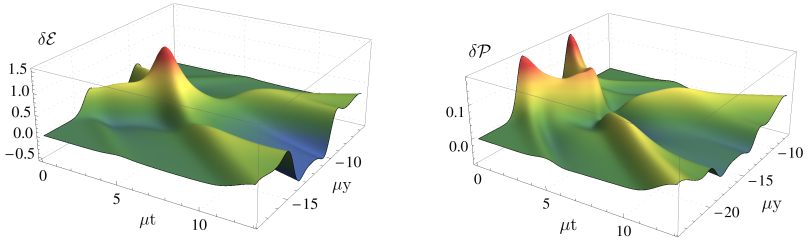




J. Casalderrey-Solana, M. Heller, D. Mateos, W. van der Schee, 1305.4919

Including transverse dynamics: inhom. shock waves

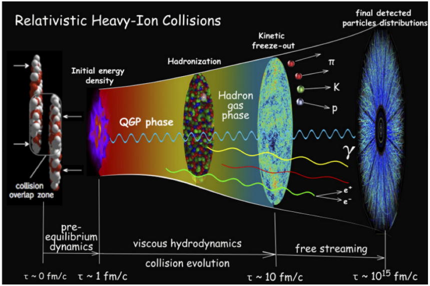
- The aim is to bring simulation closer to experiment.
- To account for elliptic flow in non-central collisions one needs dynamics in the transverse plane.
- In this simulation linearized perturbations are modelled on top of the fully non-linear, homgenous solution.

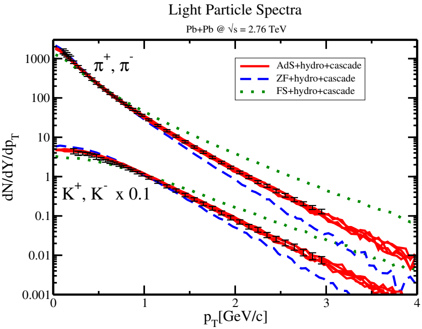


D. Fernandez, 1407.5628

Hybrid approach

- Complete simulation of a central heavy ion collision (LHC).
- Combination of AdS/CFT in the pre-equilibrium stage with hydrodynamics in the equilibrium stage and kinetic theory in the free streaming stage.





W. van der Schee, P. Romatschke, S. Pratt, 1307.2539

Plan of the talk

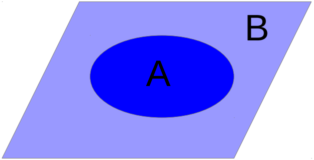
Second part: Holographic Entanglement Entropy

- Entanglement entropy
- The Ryu-Takayanagi proposal: entanglement entropy from extremal surfaces
- Geodesics on time dependent backgrounds: a glance behind the horizon?

Entanglement entropy

Definition:

- Divide a system into two parts A, B
- Reduced density matrix: $\rho_A = \mathrm{Tr}_B \rho$
- Entanglement entropy: $S_A = -\mathrm{Tr}_A \rho_A \ln \rho_A$



Properties:

- Measure for how much a quantum state is entangled.
- Entropy for observer only accessible to A: measure for quantum **information**.
- Entanglement entropy is proportional to the **degrees of freedom**.
- Can be a **quantum order parameter** in condensed matter systems.

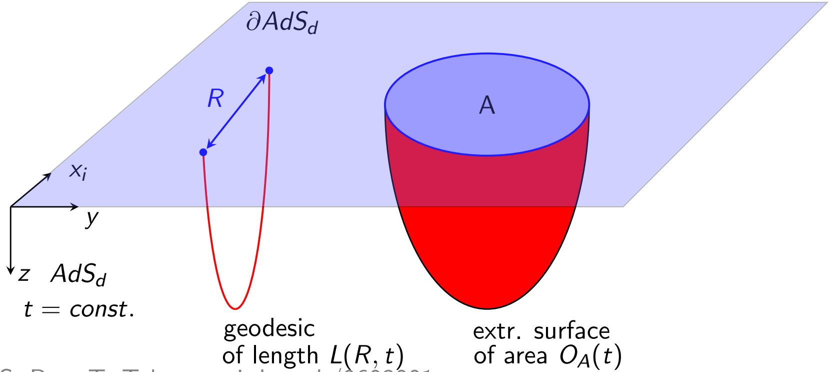
Computation in QFTs:

- In 2d-CFTs it can be done analytically (replica method).
- Universal scaling in 2d-CFTs: $S_A = \frac{c}{3} \ln \frac{l}{a}$
- In higher dimensions there is in general no analytic way - it would be nice to have a simpler method!

Holographic entanglement entropy

In QFT with a holographic dual the entanglement entropy can be computed from extremal surfaces in the gravity theory.

Ryu-Takayanagi proposal: $S_A = \frac{O_A(t)}{4G_N}$



S. Ryu, T. Takayanagi, hep-th/0603001

Questions we want to address

Is entanglement entropy a good measure for entropy production in HICs?

The plan:

- Implement a holographic thermalization model. (done - at least the simplest)
- Compute extremal surfaces. (almost there, numerics . . .)
- Compare these results to particle production in HICs. (no idea yet . . .)

Is it possible to extract information from behind a BH horizon?

Why we think it works:

- In non-stationary BH geometries, such as the homogeneous isotropization model, geodesics can reach behind the horizon. (as I will show you . . .)
- The Ryu-Takayanagi proposal relates length of these geodesics to the entanglement entropy of a region in the boundary theory.

The plan:

- Compute geometry behind the horizon. (works in our simple model)
- Compute extremal surfaces reaching behind the horizon. (works already for geodesics, as I will show you . . .)
- Use entanglement entropy to extract physical information from behind the horizon. (no idea yet . . .)

Spacelike geodesics anchored to the boundary of the anisotropic AdS_5 geometry

Geodesic equation as two-point boundary value problem (2PBVP):

$$\ddot{X}^\mu(\tau) + \Gamma^\mu_{\alpha\beta}\dot{X}^\alpha(\tau)\dot{X}^\beta(\tau) = 0, \quad BCs: X^\mu(\pm 1) = \begin{pmatrix} V(\pm 1) \\ Z(\pm 1) \\ X(\pm 1) \end{pmatrix} = \begin{pmatrix} t_0 \\ 0 \\ \pm L/2 \end{pmatrix}$$

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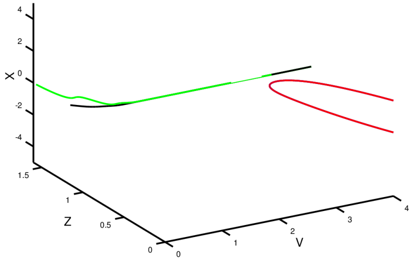

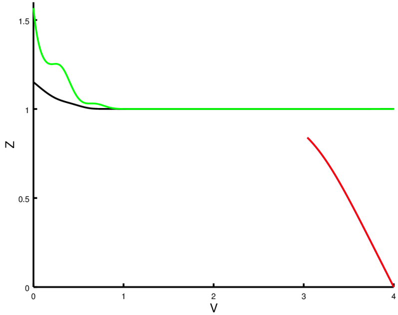

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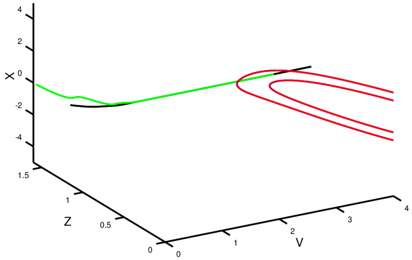

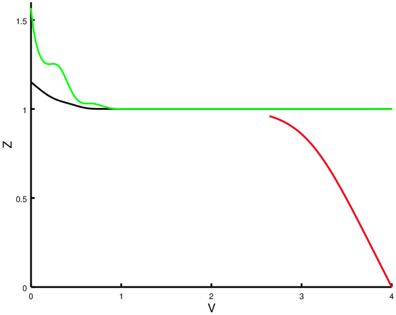

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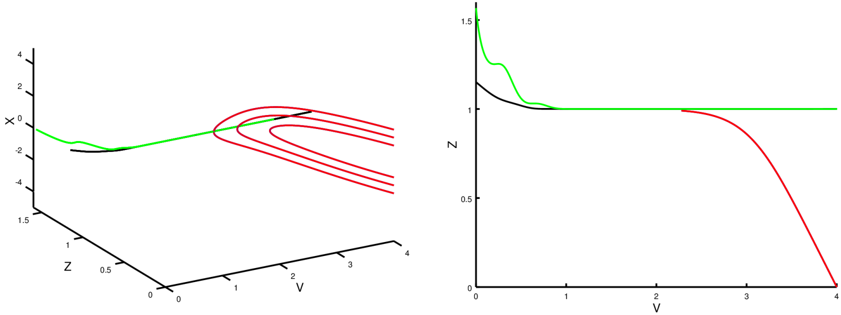

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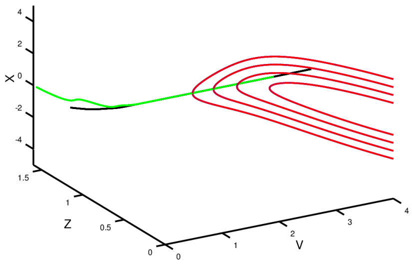

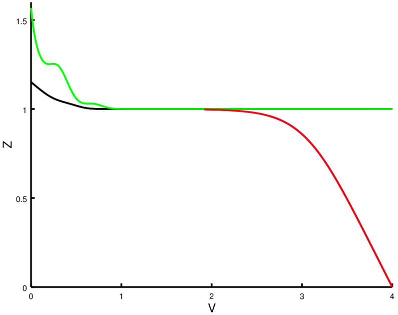

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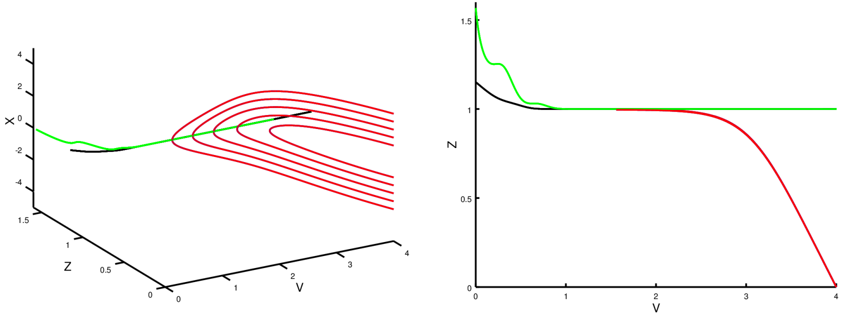

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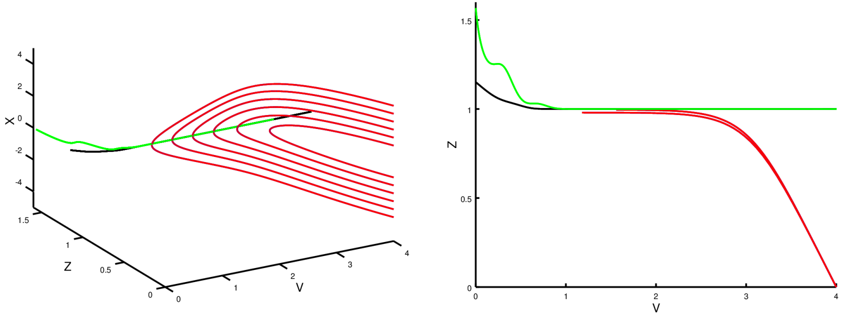

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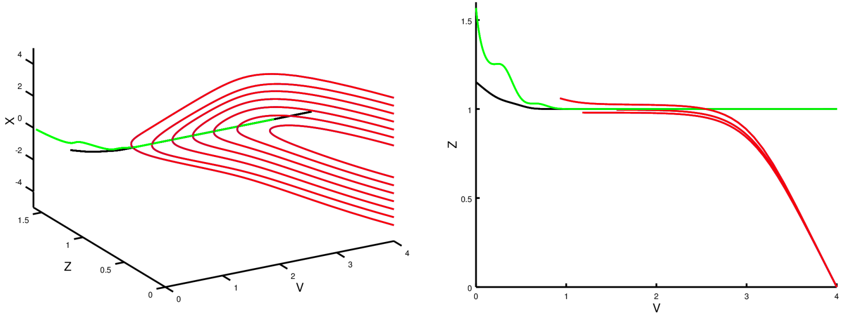

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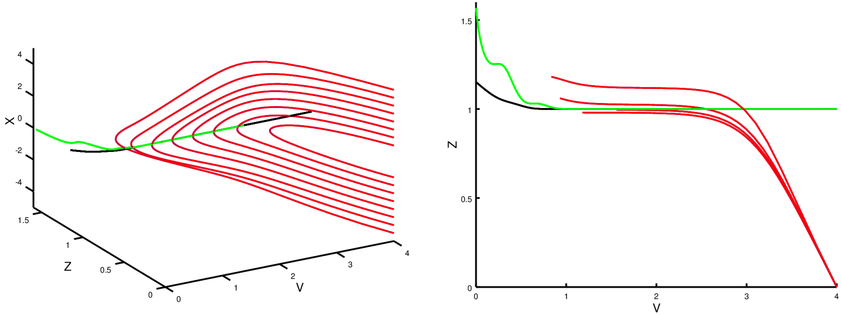

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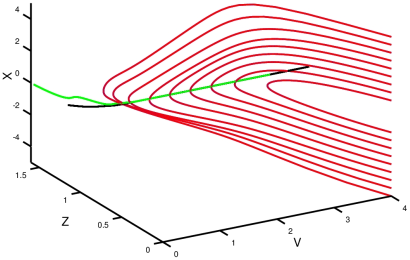

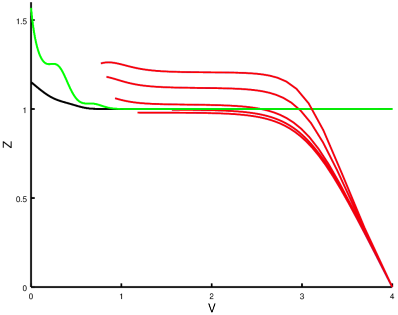

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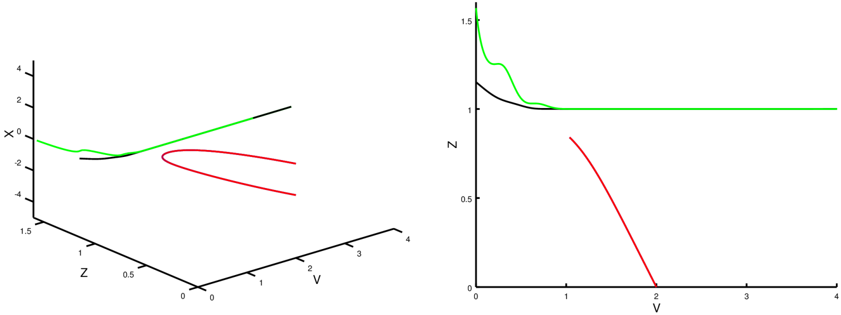

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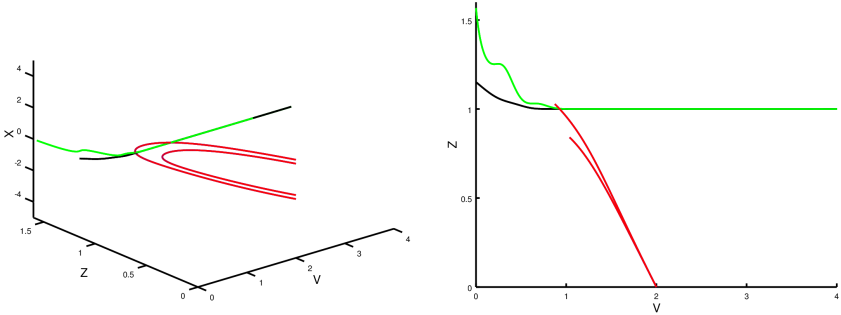

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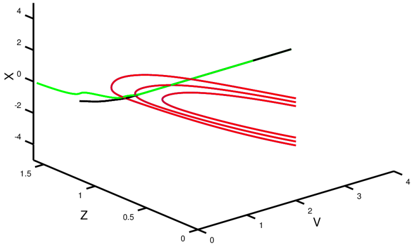

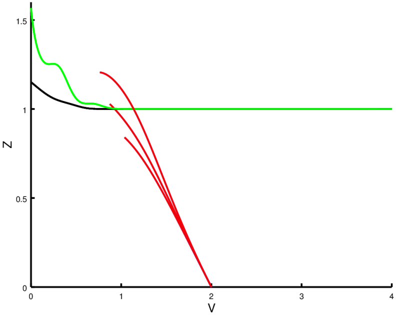

Summary & Outlook

Summary

- Black hole physics (GR) can be used to study non-equilibrium dynamics of strongly coupled gauge theories.
- Ryu-Takayanagi proposal allows to compute entanglement entropy from extremal surfaces.
- In time dependent black hole geometries geodesics can reach behind the black hole horizon. This might allows to extract information from behind the horizon.

Outlook

- We want to find out if entanglement entropy is a good measure for entropy production in HICs.
- Our ambitious aim is to use entanglement entropy to extract information form behind a black hole horizon.