Monitoring Non-Local Observables in Shock Wave Collisions

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1. Introduction

- **Heavy Ion Collisions and Quark-Gluon Plasma**
  - **Quark-gluon plasma** (QGP), a deconfined state of quarks and gluons, is produced in **heavy ion collision** at RHIC and LHC.
  - The QGP in these experiments behaves like a **strongly coupled liquid**, not like a weakly coupled gas.
  - The plasma **thermalizes on a very short time scale** of ≈10[-23] sec, which is theoretically not well understood yet.
  - Due to strong coupling **perturbative QCD is not suitable** to study the **quantum dynamics** of these collisions.
- **AdS/CFT Correspondence** [1]
  - AdS/CFT maps strongly coupled supersymmetric Yang-Mills (SYM) theory in 4D to classical gravity on 5D anti-de Sitter (AdS5) space .
  - We use SYM theory as a **toymodel** for QCD.
  - **Thermalization** in the strongly coupled 4D SYM theory is mapped to **black hole formation** in AdS5.

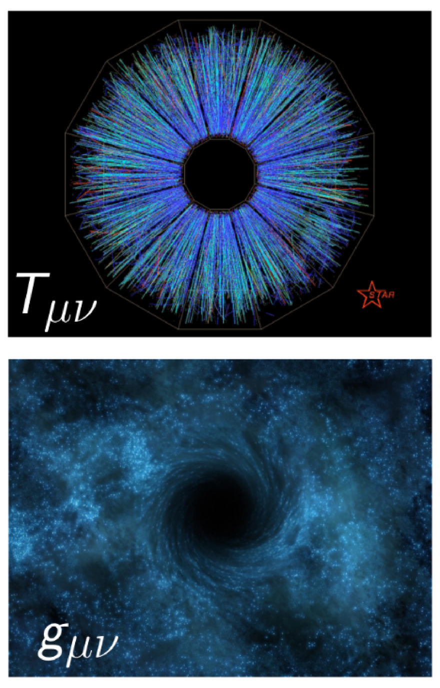

Figure 1: *AdS/CFT maps thermalization in the 4D gauge theory (top) to black hole formation in a 5D gravity theory (bottom).*

2. Local and Non-Local Observables from AdS/CFT

Within AdS/CFT it is possible to compute **expectation values of observables** in 4D SYM theory **from purely geometric objects** in the 5D gravity theory such as the **metric**, **geodesics** and **minimal surfaces**.

- **Local Observables**
  - The **energy-momentum tensor** can be extracted from the **metric** $g_{\mu\nu}$ near the boundary.
    $\langle T^{\mu\nu}(x)\rangle = -\frac{2}{\det g}\frac{\delta S}{\delta g^{\mu\nu}(x)}$
- **Non-Local Observables**
  - **Two-point functions** for gauge invariant operators $\mathcal{O}$ of large conformal weight Δ are given by the **length of geodesics** γ. [2]
    $\langle \mathcal{O}(t,\vec{x})\mathcal{O}(t,\vec{x}')\rangle \approx e^{-\Delta \mathrm{Length}(\gamma)}$
  - **Entanglement entropy** of a spatial region A is given by the **area of a minimal surface** Σ. [3]
    $S_A = -\mathrm{Tr}_A \rho_A \log \rho_A = \frac{\mathrm{Area}(\Sigma)}{4G_N}$

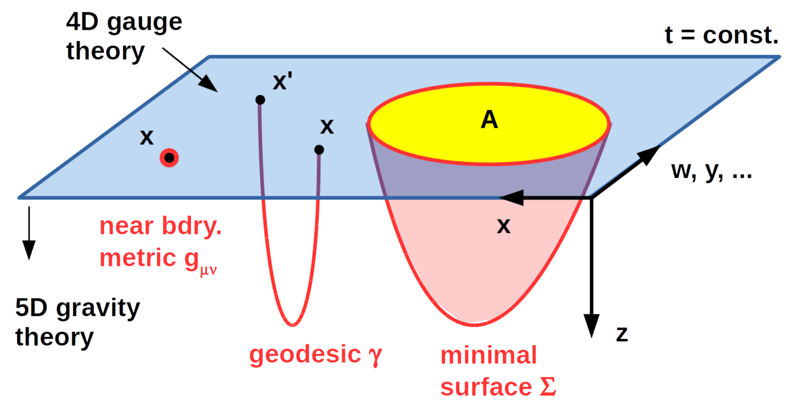

Figure 2: *Geometric description of the energy momentum tensor, two-point functions and entanglement entropy in terms of the near boundary metric, geodesics and minimal surfaces.*

3. Shock Wave Collisions in SYM theory

The **Lorentz contracted "nuclei"** in SYM are modelled as **two Gaussian energy distributions** approaching each other at the **speed of light**. The **time evolution** of the energy-momentum tensor is extracted from a **numerical relativity simulation** of **colliding gravitational shock waves** in the 5D gravity theory. [4]

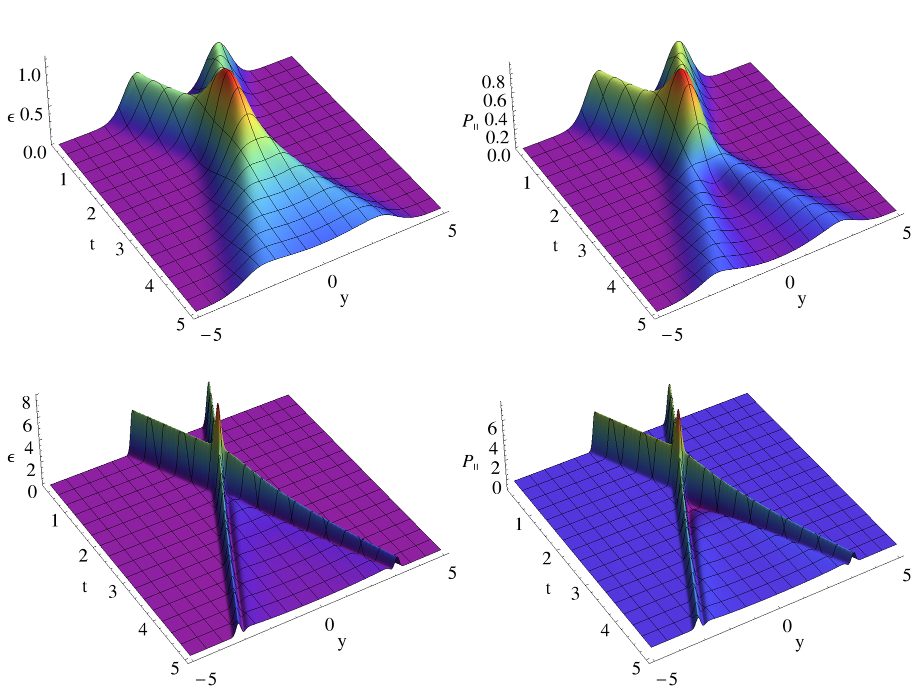

Figure 3: *Energy density $\mathcal{E}$ (left) and longitudinal pressure $\mathcal{P}_\parallel$ (right) for wide (top) and narrow shocks (bottom).*

- **Energy-Momentum Tensor**

$$\langle T^{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\begin{pmatrix} \mathcal{E} & \mathcal{S} & 0 & 0 \\ \mathcal{S} & \mathcal{P}_\parallel & 0 & 0 \\ 0 & 0 & \mathcal{P}_\perp & 0 \\ 0 & 0 & 0 & \mathcal{P}_\perp \end{pmatrix}$$

  - **Wide and narrow shocks** show **qualitatively different** behavior. [5]
- **Wide Shocks: Full-Stopping**
  - Wide shocks **stop each other** at the collision when the plasma is formed which then **explodes hydrodynamically**.
  - **Energy** and **pressure** stay **positive**.
- **Narrow Shocks: Transparency**
  - Narrow shocks **pass each other** almost **"transparently"** and the plasma is formed only after the collision.
  - **Energy** and **pressure** can be **negative** for a short time period after the collision.

4. Geodesics in AdS5

In the calculation of **two-point functions** we need the **length of spacelike geodesics** that are **attached to the boundary** at z=0 and extend into the 5D shock wave geometry.
These geodesics can be found by **numerically solving** the **geodesic equation** (1) subject to **boundary conditions** (2) that fix the endpoints at the boundary at some **spatial separation** L. [6]

$$\ddot{X}^\mu + \Gamma^\mu{}_{\alpha\beta}\dot{X}^\alpha\dot{X}^\beta = -J\dot{X}^\mu \quad (1)$$

$$X^\mu(\sigma_\pm) \equiv (V(\sigma_\pm), Z(\sigma_\pm), Y(\sigma_\pm)) = (t, z_{cut}, \pm L/2) \quad (2)$$

During the collision a **black hole horizon is formed** in the 5D shock wave geometry. The horizon has **different shape** for wide and narrow shock waves. The **geodesics tend not to cross the horizon** which leads to distortions in their **shape and length** that is **characteristic** for wide and narrow shocks.

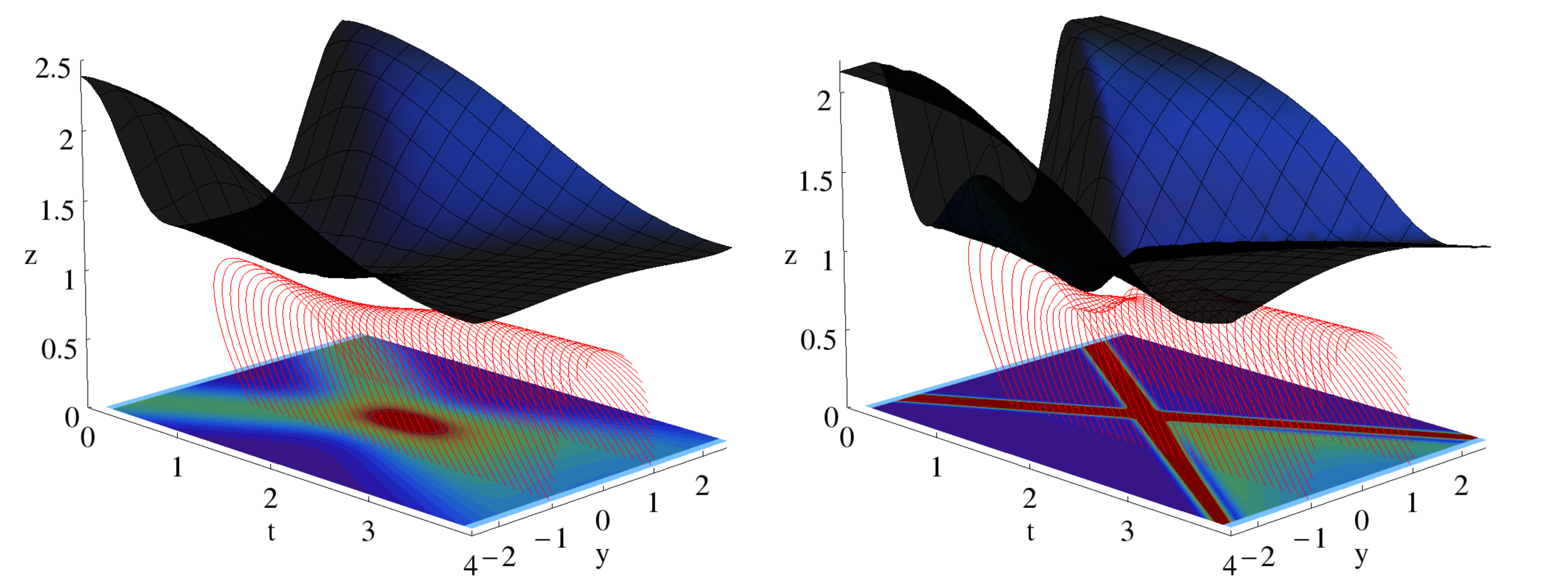

Figure 4: *Black hole horizon (black), geodesics (red) and energy contours at z=0 for wide (left) and narrow shocks (right).*

5. Two-Point Functions

Time Evolution Two-Point Functions

- The system **starts** in some **correlated state**.
- As the **shocks approach** each other **without interaction** they **destroy these initial correlations**.
- **After the collision** correlations are **restored** because of the **interactions during the collisions** new correlations are formed.
- For the **narrow shocks** these new correlations **grow significantly beyond their initial value**.
- The shock wave system follows a **top-down thermalization** pattern where **short range correlations** (small L) **reach the equilibrium first**.

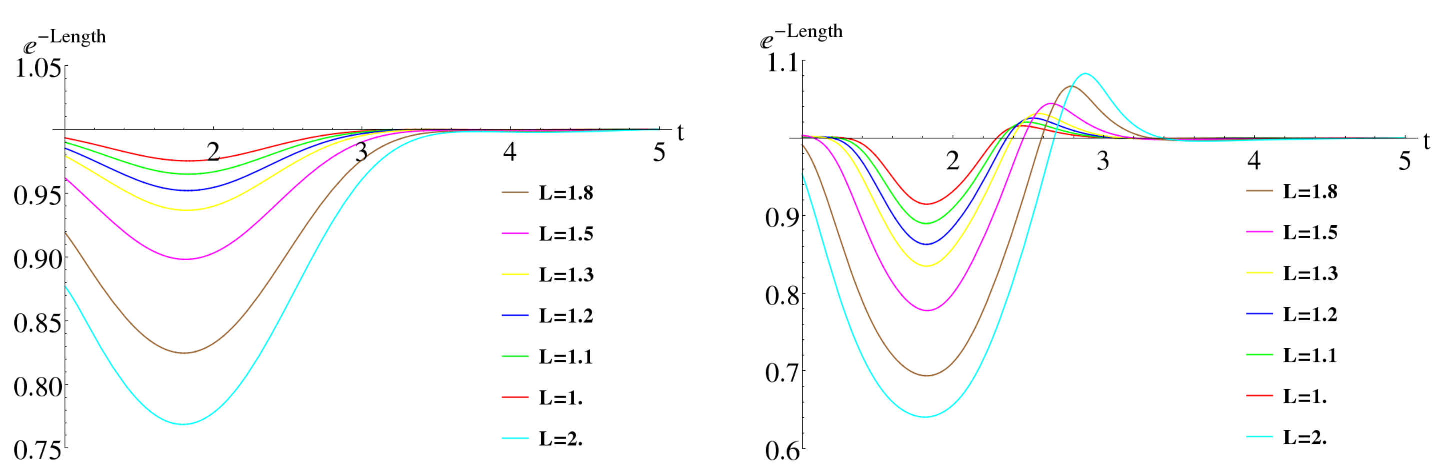

Figure 5: *Two-point function of various separations L for wide shocks (left) and narrow shocks (right).*

6. Entanglement Entropy

Time Evolution of Entanglement Entropy

- We **start with zero entanglement** by construction.
- As the **shocks enter the entangling region** the **entanglement entropy rapidly grows**.
- After the rapid initial growth follows a regime of **linear growth** which goes approx. **until the shocks collide**.
- **Right after the collision** the **wide shocks** show a **smooth fall of** whereas the **narrow shocks** have a **pronounced minimum** which is related to the **minima in the energy density** and the **longitudinal pressure**.
- As the two-point function the **entanglement entropy** shows a **top-down thermalization** pattern.

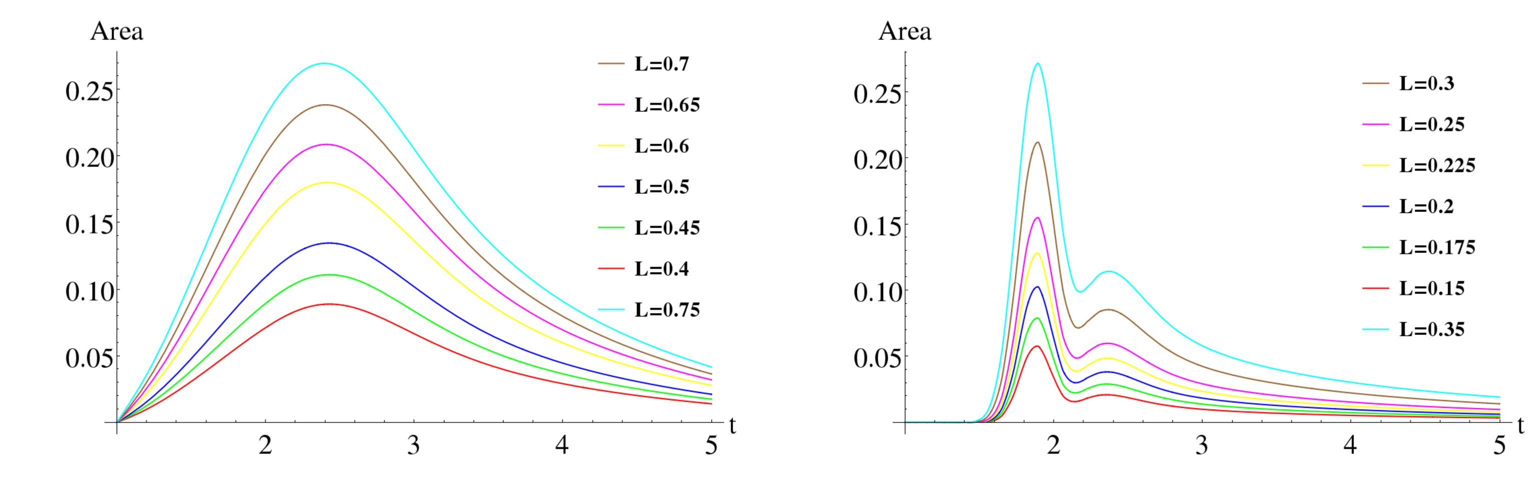

Figure 6: *Entanglement entropy of various system sizes L for wide shocks (left) and narrow shocks (right).*

7. Summary

- We use collisions of **shock waves in SYM** theory as **toymodel for real (QCD) heavy ion collisions**.
- Using the **AdS/CFT correspondence** the dynamics in these collisions can be extracted from **numerical relativity** simulations of **colliding gravitational shock waves**.
- Within AdS/CFT **non-local observables** such as **two-point functions** and **entanglement entropy** can be computed from **geodesics** and **minimal surfaces** in the gravity theory.
- From our numerical simulation we find that both, **two-point functions and entanglement entropy**, show **qualitatively different behavior for narrow and wide shocks**.
- A Mathematica code for shock wave collisions is available at Wilke van der Schees homepage: www.sites.google.com/site/wilkevanderschee/phd-thesis
- A Mathematica code for the entanglement entropy and the two-point function can be downloaded from: www.christianecker.com

8. References

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