Numerical Relativity & AdS/CFT Correspondence

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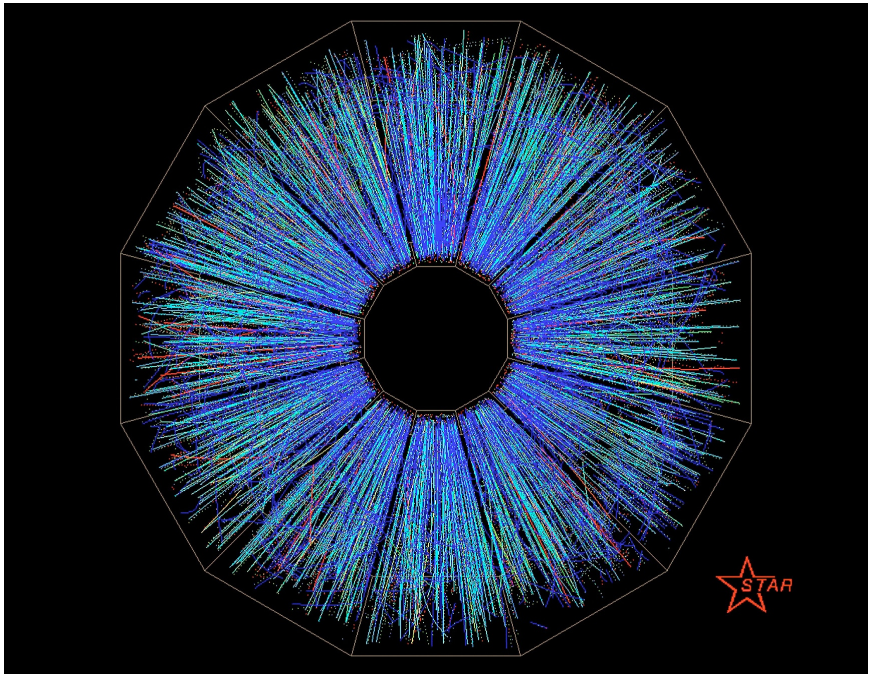
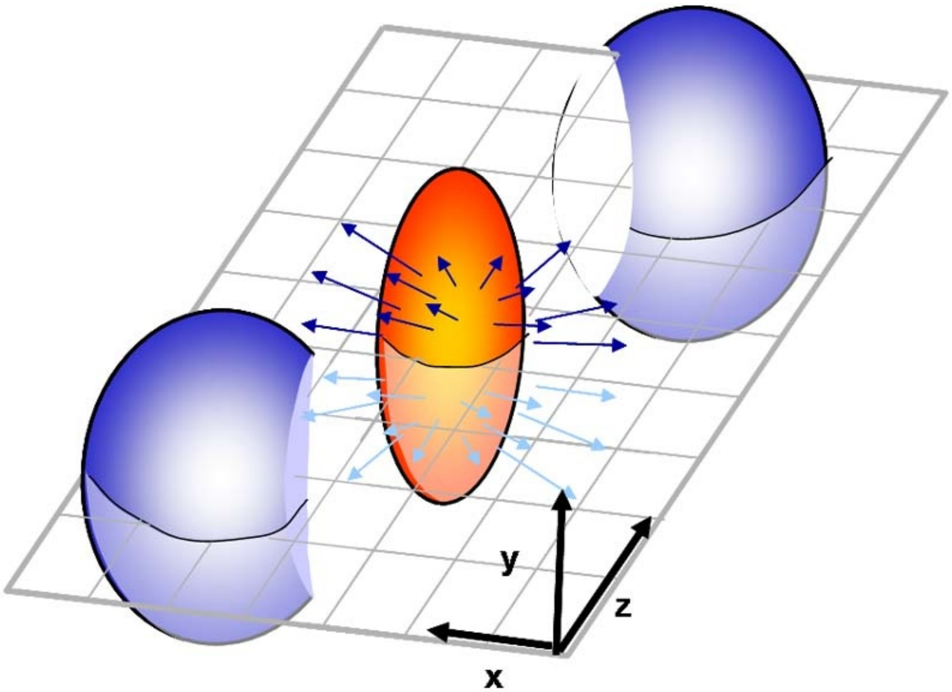
DOKTORATSKOLLEG PI $\int dk\, \Pi$ Particles and Interactions

INSTITUTE FOR THEORETICAL PHYSICS

Quark-gluon plasma in heavy ion collisions

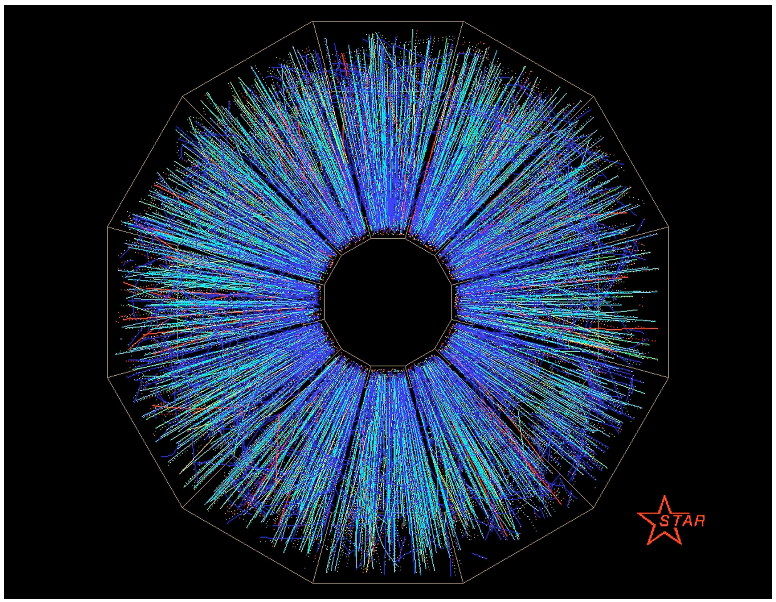
Quark-gluon plasma (QGP) is a **deconfined phase of quarks and gluons** produced in **heavy ion collision** (HIC) experiments at **RHIC** and **LHC**.

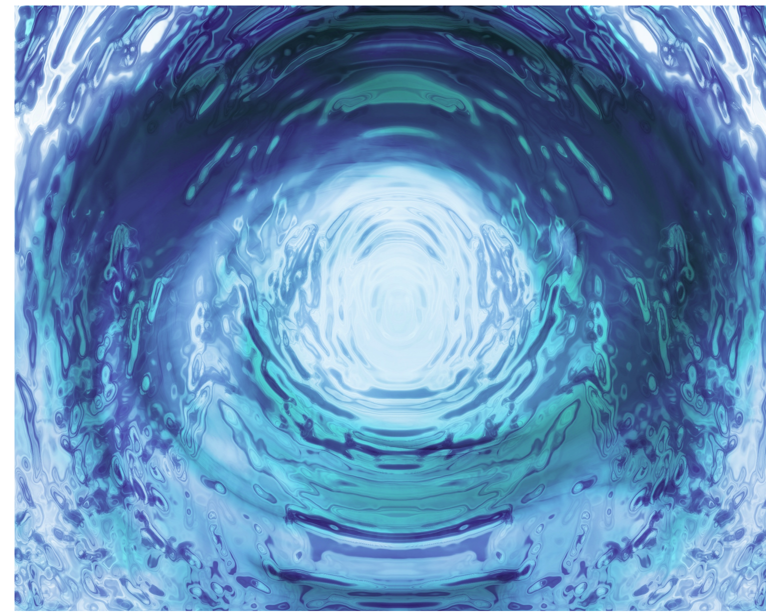




Why AdS/CFT?

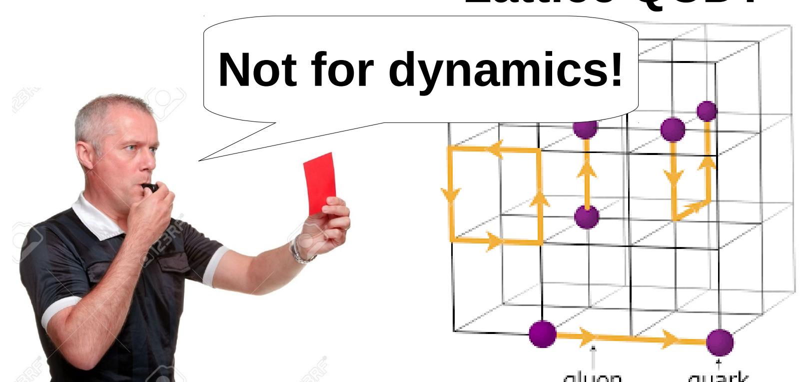
The QGP produced in HIC's behaves like a **strongly coupled liquid** rather than a **weakly coupled gas.**





Perturbative QCD?

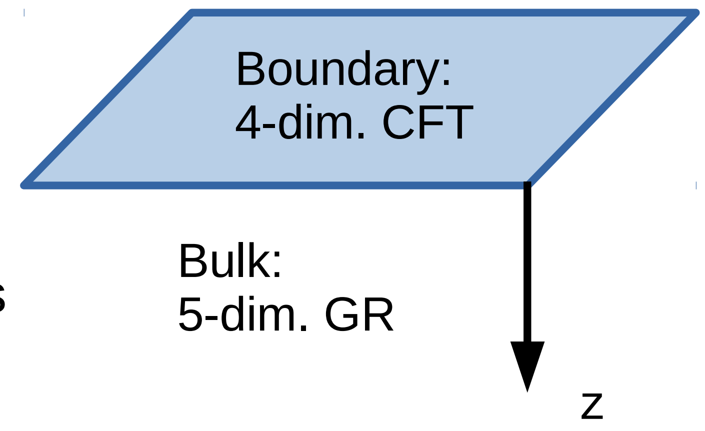
Lattice QCD?



AdS/CFT correspondence

<u>AdS/CFT correspondence:</u> [Maldacena 97]

Type IIB string theory on $AdS_5 \times S^5$ is equivalent to $\mathcal{N}=4$ super symmetric $SU(N_c)$ **Yang-Mills theory** in 4D.



<u>Supergravity limit:</u>

Strongly coupled large N_c $\mathcal{N}=4$ $SU(N_c)$ SYM theory is equivalent to **classical supergravity** on AdS_5.

<u>Strategy:</u>

- Use $\mathcal{N}=4$ SYM as **toymodel** for **QCD** in the strongly coupled regime.
- Build a **gravity model** dual to HICs, like colliding **gravitational shock waves**.
- Switch on the computer and solve the 5-dim. gravity problem **numerically**.
- Use the **holographic dictionary** to compute **observables in the 4 dim. field theory** form those gravity result.

Solving the Einstein equations on asymptotically AdS

We want to solve the **5 dim. vacuum Einstein equations** (EE) with **negative cosmological constant** Λ.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

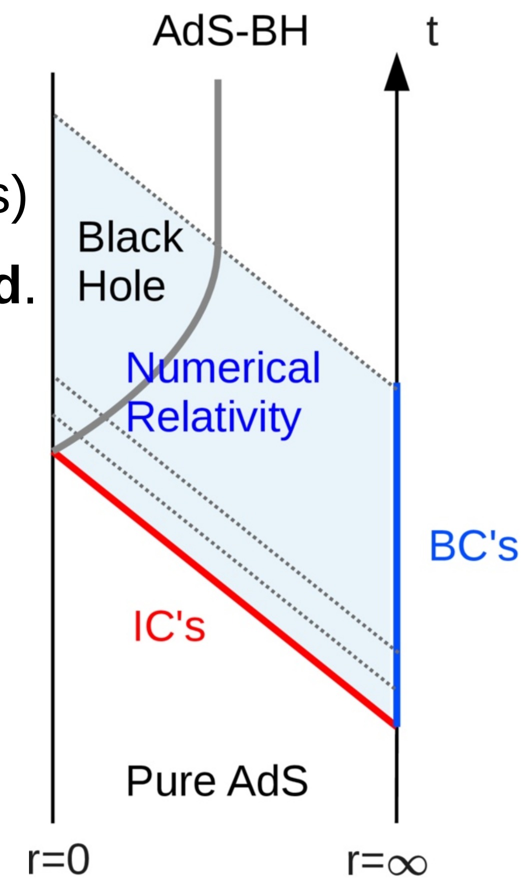
Eddington-Finkelstein gauge (light-like slicing) decouples the EE into a **nested set of ODEs**. (method of characteristics)

These ODEs can be efficiently solved with a **spectral method**.

The **time evolution** is done with a **4$^{\text{th}}$ order Runge-Kutta method** (RK4).

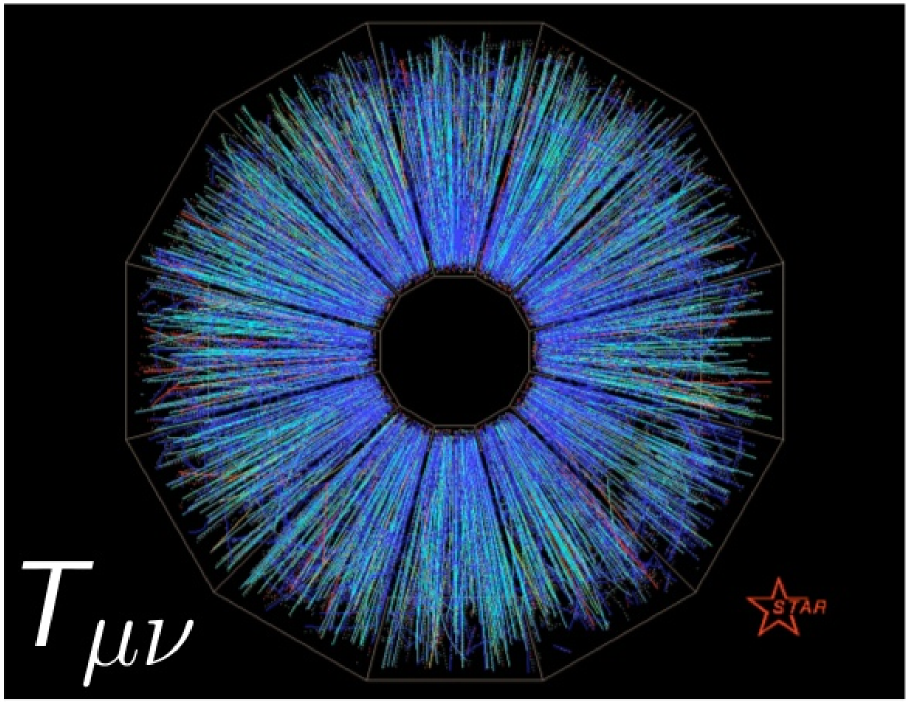
AdS is **not globally hyperbolic** – need **IC's & BC's** to formulate a **well defined initial value problem** (IVP).

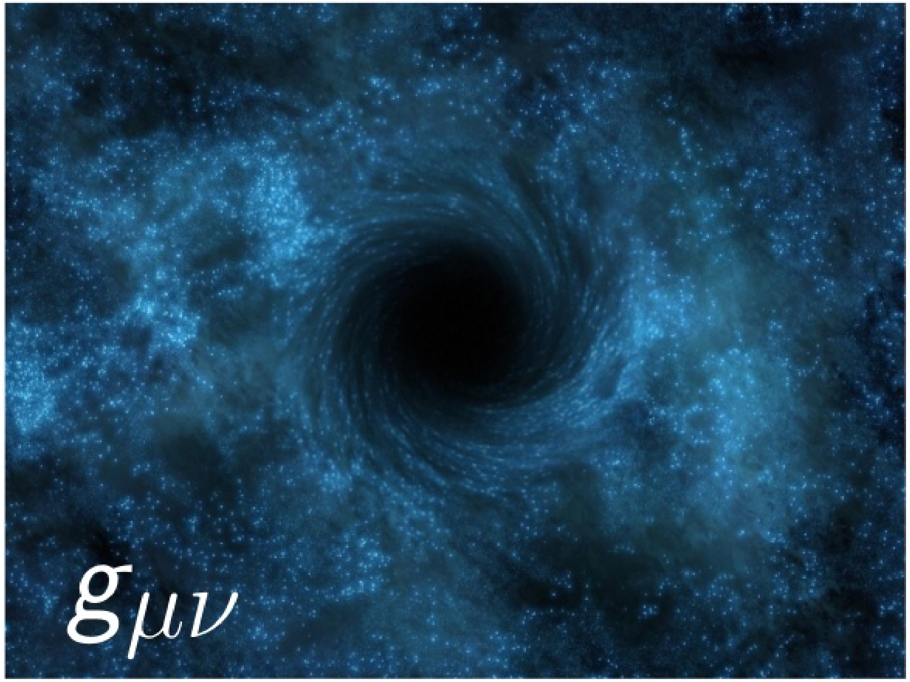
- **BC's:** boundary metric is **4-dim Minkovski**
  = **background metric** of the boundary **QFT**
- **IC's: two gravitational shock waves** in AdS
  = Lorentz contracted **nuclei in the QFT**



Holographic thermalization

Thermalization = Black hole formation





Entanglement entropy

Divide the system into **two parts** A,B.
The total Hilbert space factorizes:

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

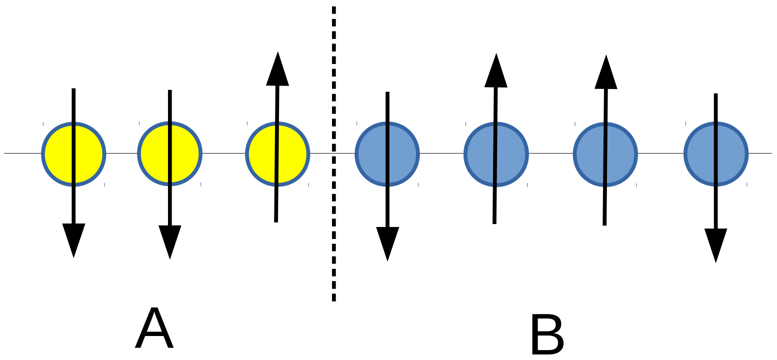
The **reduced density matrix** of A is obtained by the trace over $\mathcal{H}_\mathcal{B}$

$$\rho_A = \mathrm{Tr}_B \rho$$

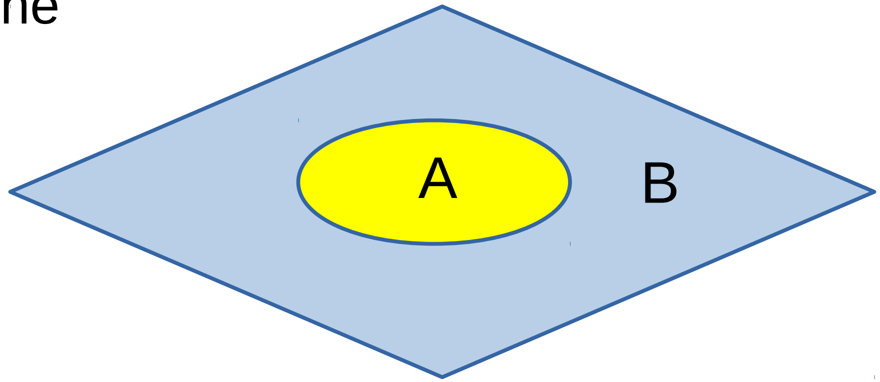
Entanglement entropy is defined as the **von Neumann entropy** of ρ_A:

$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$

lattice model



quantum field theory



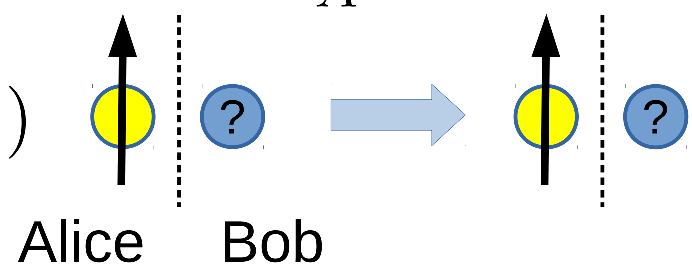
Entanglement entropy in a two quantum bit system

Consider a quantum system of two spin 1/2 dof's.
Observer Alice has only access to one spin and Bob to the other spin.

A **product state (not entangled)** in a two spin 1/2 system:

$$S_A = 0$$

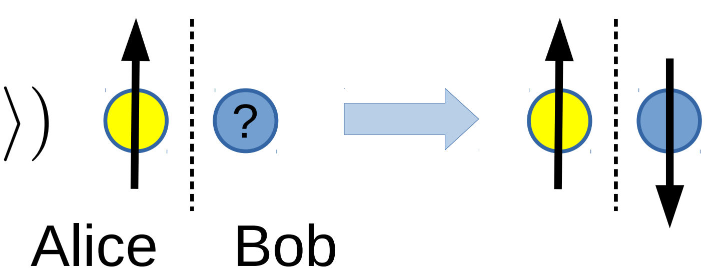
$$|\psi\rangle = \frac{1}{2}(|\uparrow_A\rangle + \cancel{|\downarrow_A\rangle}) \otimes (|\uparrow_B\rangle + |\downarrow_B\rangle)$$



A (maximally) **entangled state** in a two spin 1/2 system:

$$S_A = \log 2$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A\rangle \otimes |\downarrow_B\rangle - \cancel{|\downarrow_A\rangle \otimes |\uparrow_B\rangle})$$



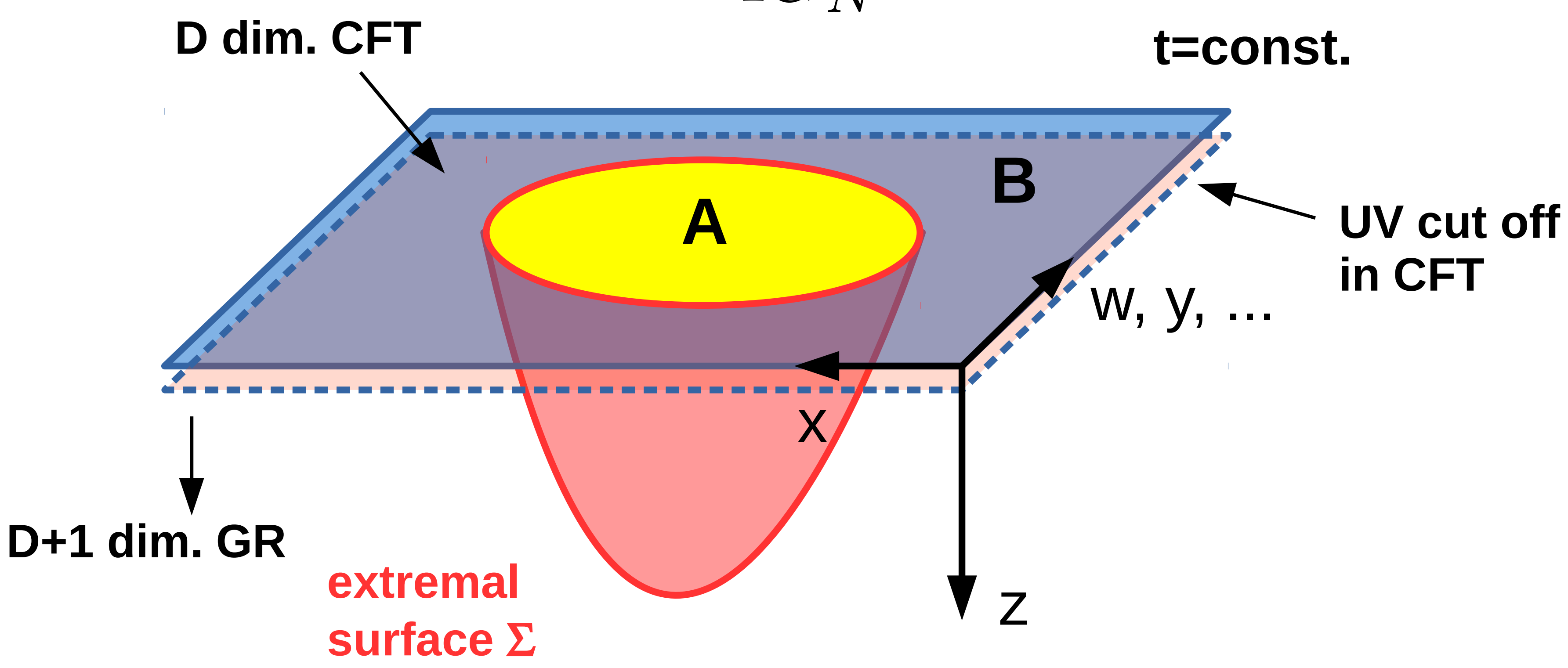
Entanglement entropy is a **measure** for **entanglement** in a quantum system.

Holographic entanglement entropy

Within **AdS/CFT** entanglement entropy can be computed form the **area of minimal (extremal) surfaces** in the gravity theory.

$$S_A = \frac{\text{Area}(\Sigma)}{4G_N}$$

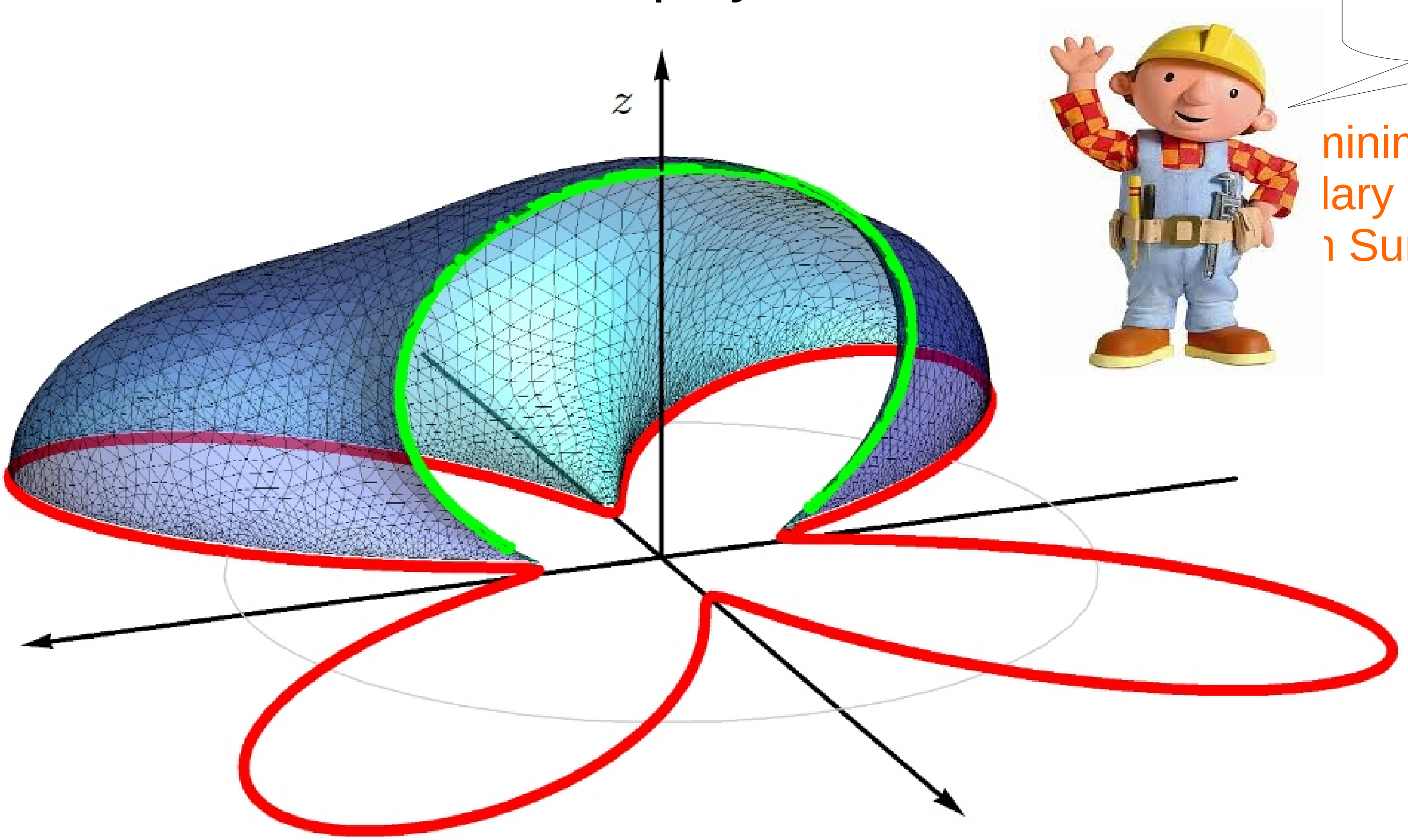
[Ryu-Takayanagi 06, Hubeny-Rangamani-Takayanagi 07]



Holographic entanglement entropy

- In practice computing extremal co-dim. 2 hyper-surfaces is numerically involved. [work in progress: CE-Grumiller-Khavari]
- Can we somehow simplify our lives?

Yes we can!

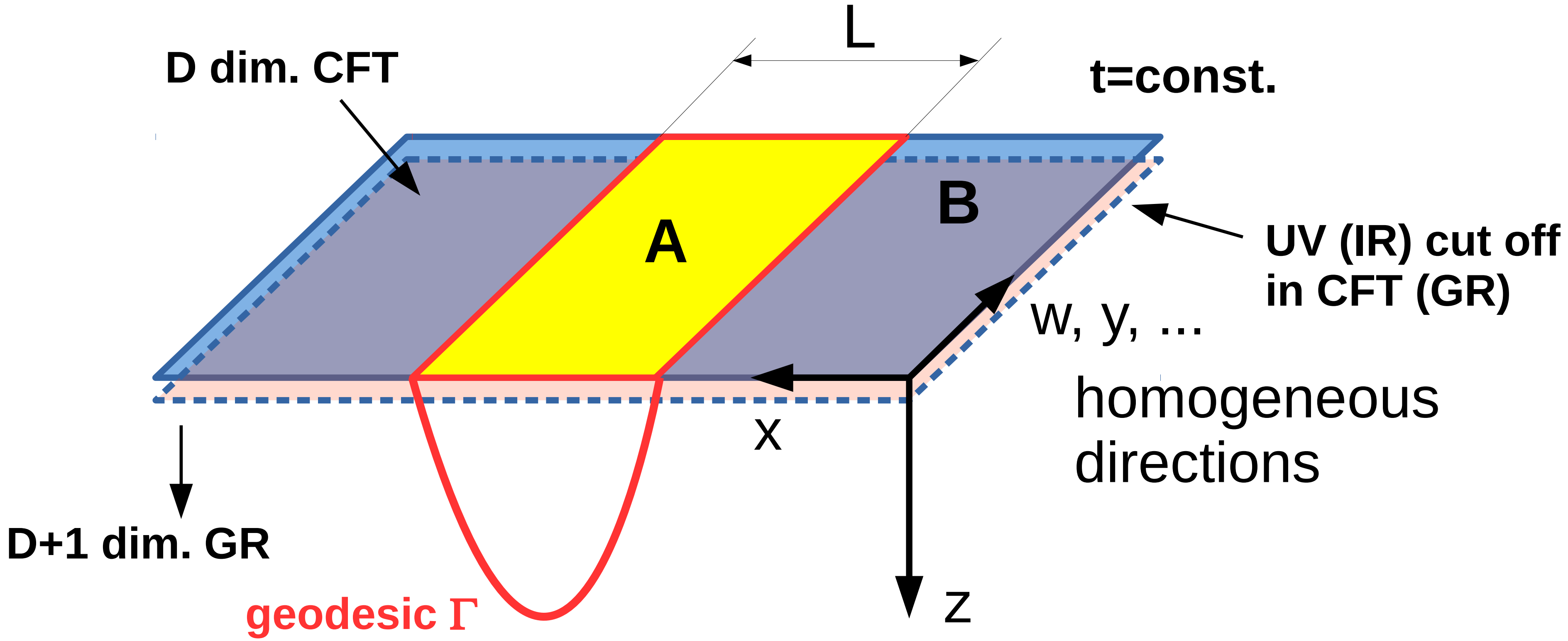


ninimal surface for a star
lary region (red) in AdS4
ı Surface Evolver]

Entanglement entropy from geodesics

Consider a **stripe region** of **infinite extend** in **homogeneous directions** of the geometry.
The **entanglement entropy** is prop. to the **geodesics length** in an **auxiliary spacetime**.

$$S_A = const.\frac{\mathrm{Length}(\Gamma)}{4G_N} \qquad \tilde{g}_{\mu\nu} = \Omega(z,t,x)^2 g_{\mu\nu}$$

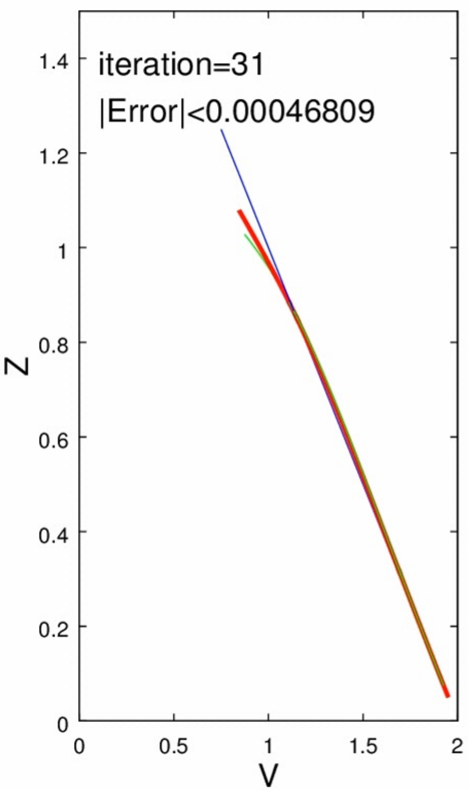


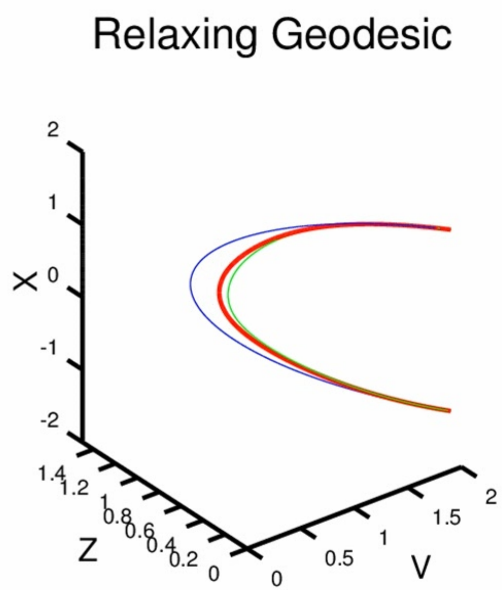
Numerics: relax, don't shoot!

Geodesic equation as **two point boundary value problem**.

$$\ddot{X}^{\mu}(\tau) + \Gamma^{\mu}_{\alpha\beta}\dot{X}^{\alpha}(\tau)\dot{X}^{\beta}(\tau) = 0$$

$$BCs: \quad (V(\pm 1), Z(\pm 1), X(\pm 1)) = (t_0, 0, L/2)$$





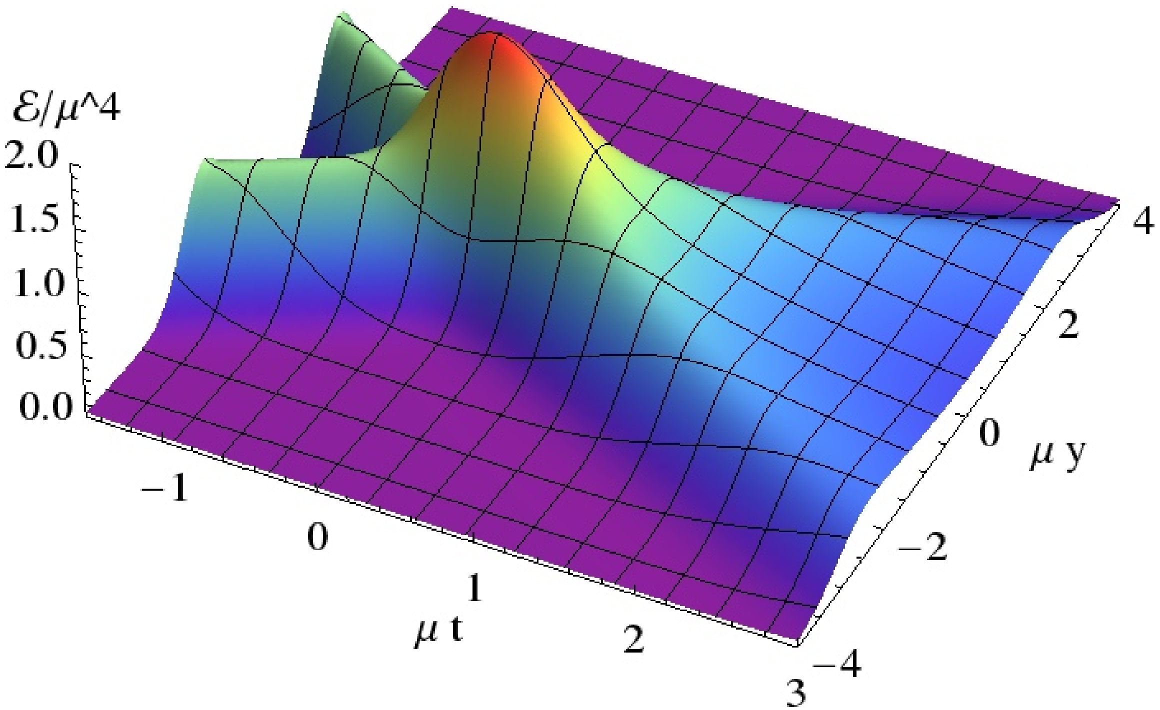
- There are two **standard numerical methods** for solving two point boundary value problems. [see Numerical Recipes]

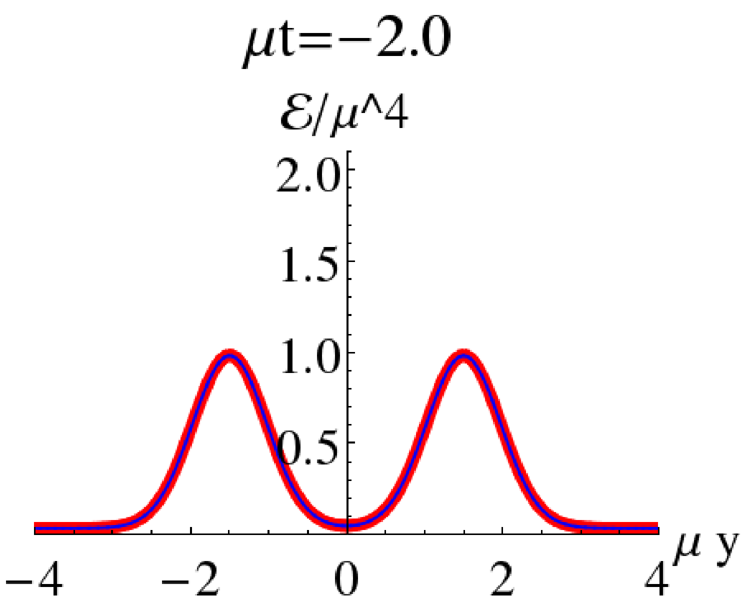
Shooting:
Very **sensitive to initialization** on **asymptotic AdS** spacetimes.

Relaxation:
Converges very **fast** if **good initial guess** is provided.

Holographic shock wave collisions

HIC is modeled by **two colliding sheets of energy** with **infinite extend in transverse direction** and **Gaussian profile** in **beam direction**. [Chesler-Yaffe 10]



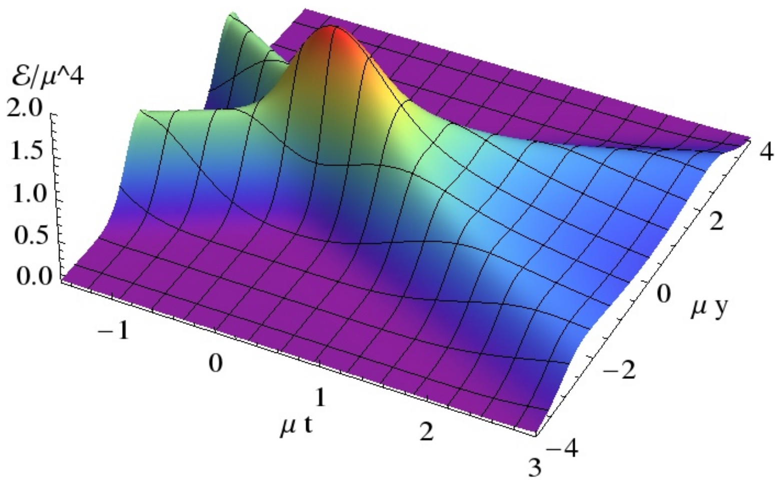


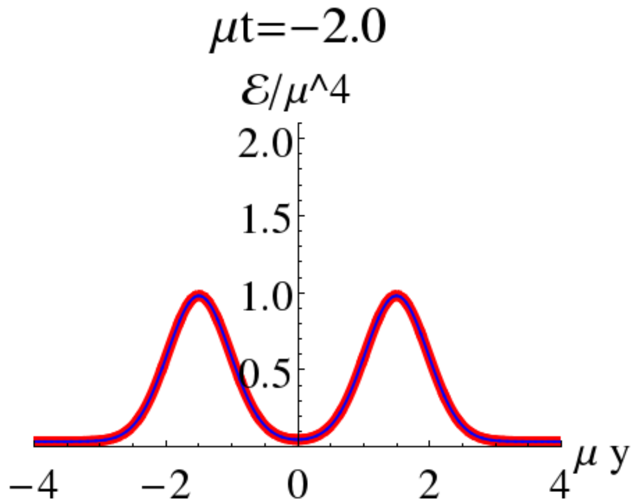
Wide vs. narrow shocks

Two qualitatively different dynamical regimes

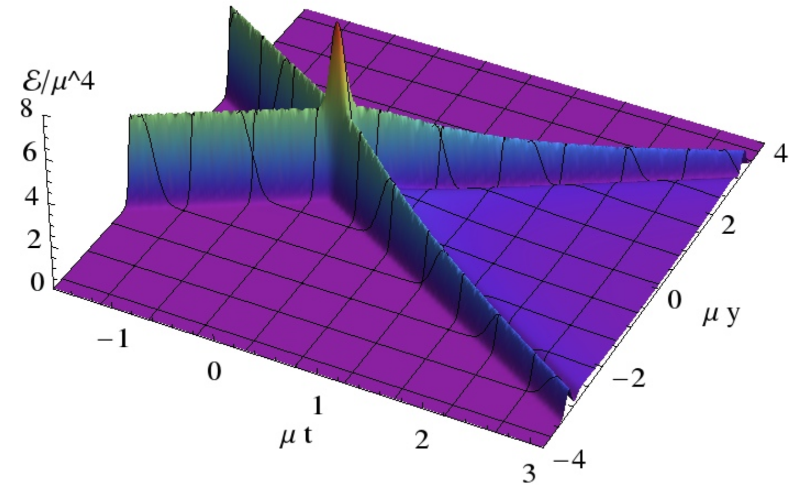
[Solana-Heller-Mateos-van der Schee 12]

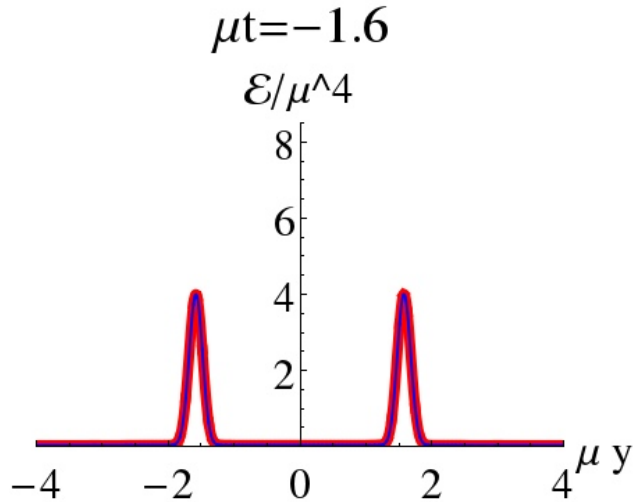
- **Wide** shocks (~RHIC): **full stopping**



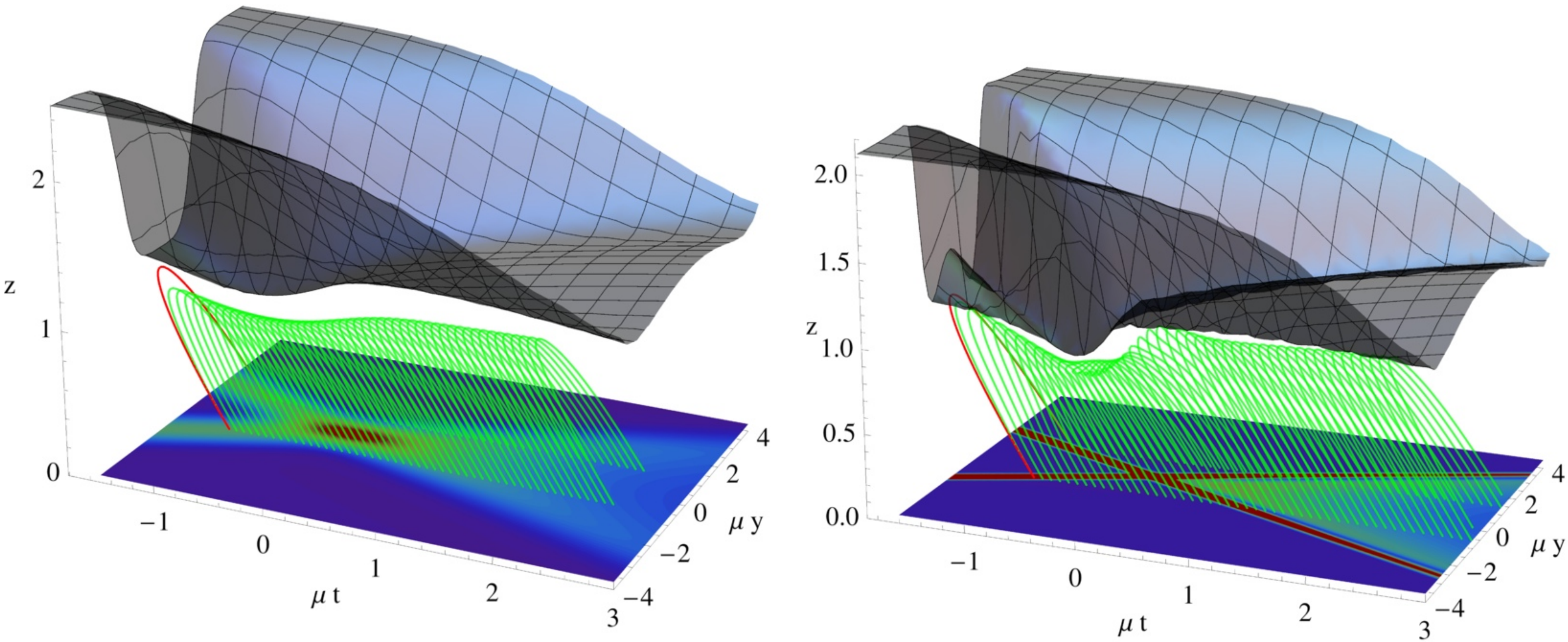


- **Narrow** shocks (~LHC): **transparency**



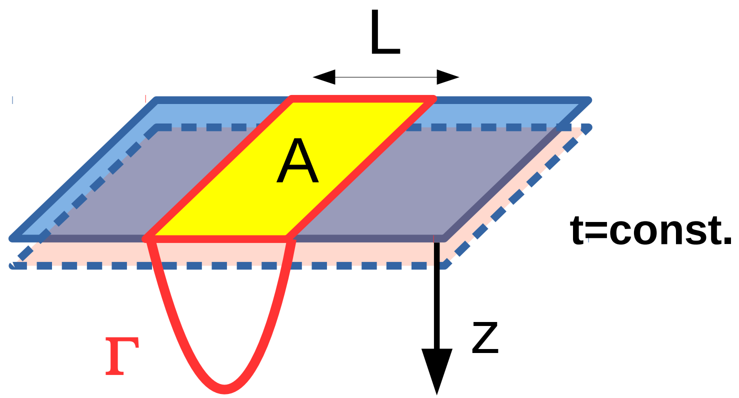


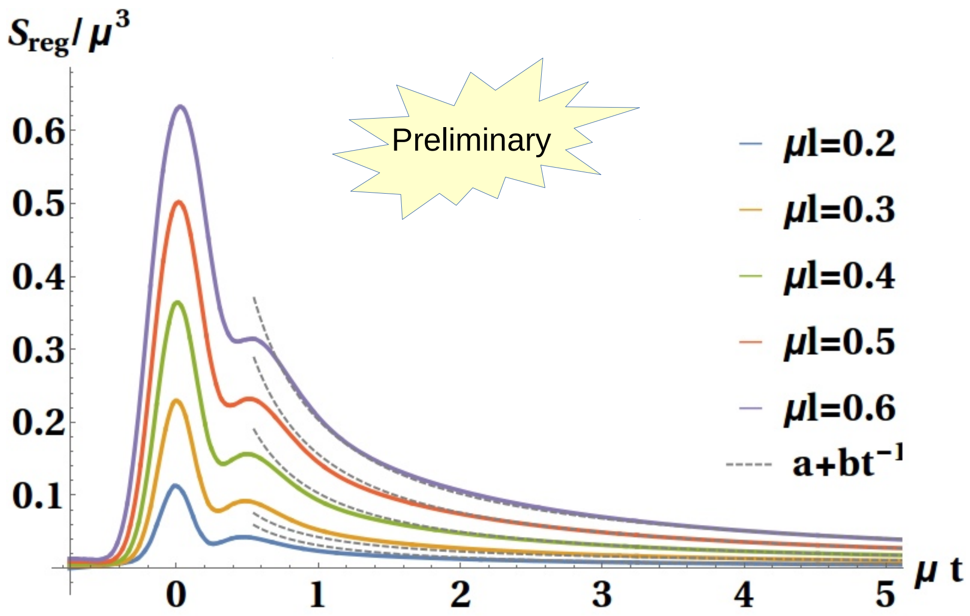
Geodesics and apparent horizon

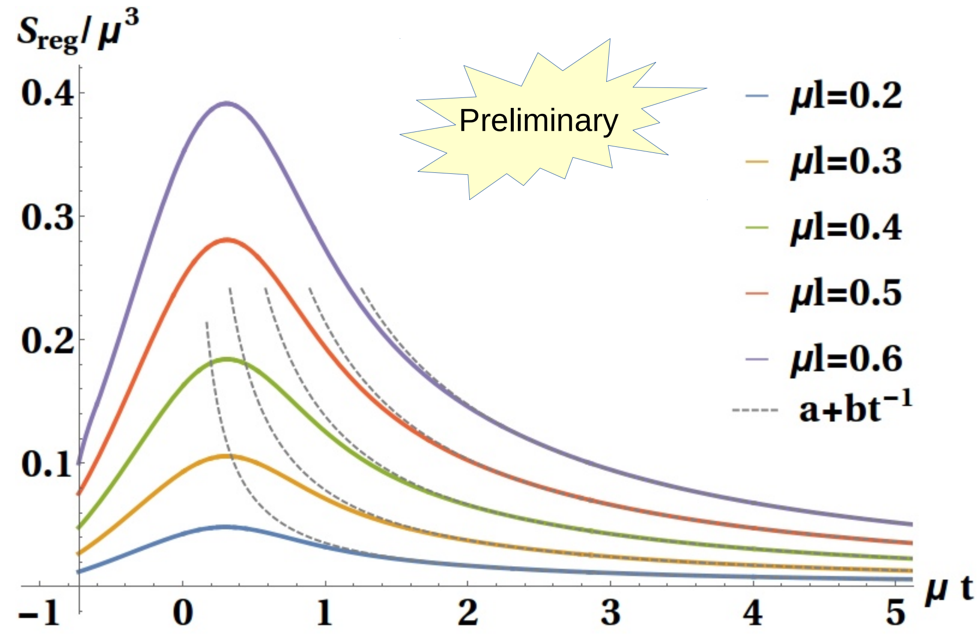


Entanglement entropy

$$S_A = const.\frac{\text{Length}(\Gamma)}{4G_N}$$

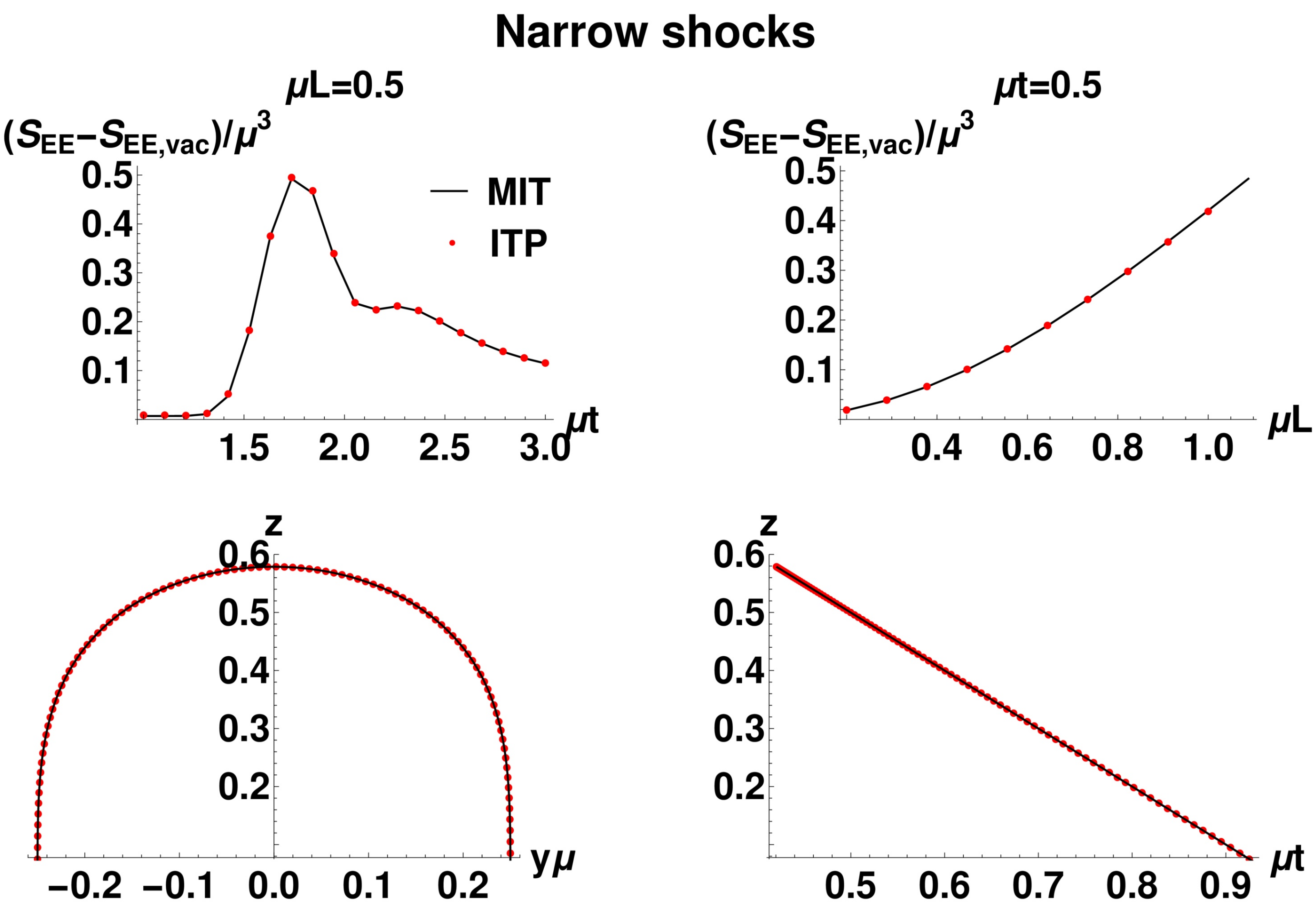






[CE-Grumiller-Van der Schee-Stanzer-Stricker 16XX.XXXXX]

Is the numerics right?



Gridsize	S_{reg}
50	0.0794411
80	0.112574
100	0.114726
200	0.115453
300	0.115452
400	0.115451

Summary

- **AdS/CFT** allows to study the **real time dynamics** of **strongly coupled QFT's** by solving the IVP of (classical) **supergravity** theories.
- **Entanglement entropy** may serve as an **order parameter** for the **full stopping–transparency transition** in holographic shock wave collisions. [CE-Grumiller-Van der Schee-Stanzer-Stricker 16XX.XXXXX]

Work in progress

- **Going beyond supergravity**: string corrections, semi-holography, …
  [CE-Mukhopadhyay-Preiss-Rebhan-Stricker]
- On the field theory side the **null energy condition** (NEC) is **violated** in narrow shock wave collisions. The **quantum null energy condition** (QNEC) is conjectured to give an upper bound for this violation.

$$\langle T_{kk} \rangle \geq S''$$

[CE-Grumiller-Van der Schee-Stanzer]